

Bài 3. Công thức lượng giác

Thời gian làm bài: 40 phút (Không kể thời gian giao đề)

Họ tên thí sinh:

Số báo danh:

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Câu 1. Cho biết $\sin \alpha = \frac{1}{3}$ và $\frac{\pi}{2} < \alpha < \pi$. Khi đó:

*a) $\cos \alpha = -\frac{2\sqrt{2}}{3}$

*b) $\sin 2\alpha = -\frac{4\sqrt{2}}{9}$

*c) $\cos 2\alpha = \frac{7}{9}$

d) $\cot 2\alpha = \frac{7\sqrt{2}}{8}$

Lời giải

a) Đúng	b) Đúng	c) Đúng	d) Sai
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Vì $\frac{\pi}{2} < \alpha < \pi$ nên $\cos \alpha < 0$.

Ta có: $\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \frac{1}{9} = \frac{8}{9} \Rightarrow \cos \alpha = -\frac{2\sqrt{2}}{3}$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = 2 \cdot \frac{1}{3} \cdot \left(-\frac{2\sqrt{2}}{3}\right) = -\frac{4\sqrt{2}}{9}$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha = 1 - 2 \left(\frac{1}{3}\right)^2 = \frac{7}{9}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = -\frac{4\sqrt{2}}{7}; \cot 2\alpha = \frac{1}{\tan 2\alpha} = -\frac{7\sqrt{2}}{8}$$

Câu 2. Cho biết $\cos 2\alpha = -\frac{1}{4}$ và $\pi < \alpha < \frac{3\pi}{2}$. Khi đó:

*a) $\sin \alpha < 0, \cos \alpha < 0$

b) $\sin \alpha = \frac{\sqrt{10}}{4}$

c) $\cos \alpha = \frac{\sqrt{6}}{4}$

*d) $\cot \alpha = \frac{\sqrt{15}}{5}$

Lời giải

a) Đúng	b) Sai	c) Sai	d) Đúng
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Vì $\pi < \alpha < \frac{3\pi}{2}$ nên $\sin \alpha < 0, \cos \alpha < 0$.

Ta có: $\cos 2\alpha = -\frac{1}{4} \Rightarrow 1 - 2 \sin^2 \alpha = -\frac{1}{4} \Rightarrow \sin^2 \alpha = \frac{5}{8} \Rightarrow \sin \alpha = -\frac{\sqrt{10}}{4}$;

$$\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \frac{10}{16}} = -\frac{\sqrt{6}}{4};$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\sqrt{15}}{3}$$

$$\cot \alpha = \frac{1}{\tan \alpha} = \frac{\sqrt{15}}{5}$$

Câu 3. Cho biết $\sin \alpha = \frac{3}{5}, \frac{\pi}{2} < \alpha < \pi$. Khi đó:

*a) $\cos \alpha < 0$

c) $\tan \alpha = \frac{3}{4}$

Lời giải

a) Đúng	b) Đúng	c) Sai	d) Sai
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Vì $\frac{\pi}{2} < \alpha < \pi \Rightarrow \cos \alpha < 0$.

Ta có $\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = -\frac{4}{5}$

$\Rightarrow \tan \alpha = \frac{\sin \alpha}{\cos \alpha} = -\frac{3}{4}$.

$$\tan\left(\alpha + \frac{\pi}{3}\right) = \frac{\tan \alpha + \tan \frac{\pi}{3}}{1 - \tan \alpha \tan \frac{\pi}{3}} = \frac{\tan \alpha + \sqrt{3}}{1 - \sqrt{3} \tan \alpha}$$

Ta có:

$$\tan\left(\alpha + \frac{\pi}{3}\right) = \frac{-\frac{3}{4} + \sqrt{3}}{1 - \sqrt{3}\left(-\frac{3}{4}\right)} = \frac{-3 + 4\sqrt{3}}{4 + 3\sqrt{3}} = \frac{48 - 25\sqrt{3}}{11}$$

Suy ra:

Câu 4. Cho biết $\sin \alpha = -\frac{12}{13}, \frac{3\pi}{2} < \alpha < 2\pi$. Khi đó:

*a) $\cos \alpha > 0$

*c) $\tan \alpha = -\frac{12}{5}$

*b) $\cos \alpha = \frac{5}{13}$

d) $\cos\left(\frac{\pi}{3} - \alpha\right) = \frac{5 - \sqrt{3}}{26}$

Lời giải

a) Đúng	b) Đúng	c) Đúng	d) Sai
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Ta có: $\cos\left(\frac{\pi}{3} - \alpha\right) = \cos \frac{\pi}{3} \cos \alpha + \sin \frac{\pi}{3} \sin \alpha$

$$= \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \cdot \left(-\frac{12}{13}\right) = \frac{1}{2} \cos \alpha - \frac{6\sqrt{3}}{13}.$$

Vì $\frac{3\pi}{2} < \alpha < 2\pi \Rightarrow \cos \alpha > 0$

Ta có $\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \frac{5}{13}$.

Suy ra: $\cos\left(\frac{\pi}{3} - \alpha\right) = \frac{1}{2} \cdot \frac{5}{13} - \frac{6\sqrt{3}}{13} = \frac{5 - 12\sqrt{3}}{26}$

Câu 5. Cho biết $\sin x = \frac{1}{\sqrt{3}}$ và $0 < x < \frac{\pi}{2}$; khi đó:

*a) $\cos x > 0$

c) $\tan x = \frac{\sqrt{3}}{3}$

*b) $\cos x = \frac{\sqrt{6}}{3}$

d) $\cos\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{6} - 3}{8}$

Lời giải

a) Đúng	b) Đúng	c) Sai	d) Sai
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a) Vì $0 < x < \frac{\pi}{2}$ nên $\cos x > 0$.

Ta có: $\sin x = \frac{1}{\sqrt{3}} \Rightarrow \cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \frac{1}{3}} = \frac{\sqrt{6}}{3}$

$\cos\left(x + \frac{\pi}{3}\right) = \cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3} = \frac{\sqrt{6}}{3} \cdot \frac{1}{2} - \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6} - 3}{6}$

Câu 6. Cho biết $\cos x = -\frac{12}{13}$ và $\pi < x < \frac{3\pi}{2}$; khi đó:

a) $\sin x > 0$

c) $\cot x = \frac{5}{12}$

*b) $\sin x = -\frac{5}{13}$

*d) $\sin\left(\frac{\pi}{3} - x\right) = \frac{5 - 12\sqrt{3}}{26}$

Lời giải

a) Sai	b) Đúng	c) Sai	d) Đúng
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Vì $\pi < x < \frac{3\pi}{2}$ nên $\sin x < 0$.

Ta có: $\cos x = -\frac{12}{13} \Rightarrow \sin x = -\sqrt{1 - \cos^2 x} = -\sqrt{1 - \left(-\frac{12}{13}\right)^2} = -\frac{5}{13}$

$\sin\left(\frac{\pi}{3} - x\right) = \sin \frac{\pi}{3} \cos x - \cos \frac{\pi}{3} \sin x = \frac{\sqrt{3}}{2} \left(-\frac{12}{13}\right) - \frac{1}{2} \left(-\frac{5}{13}\right) = \frac{5 - 12\sqrt{3}}{26}$

Câu 7. Cho biết $\tan x = \sqrt{2}$ và $0 < x < 90^\circ$. Khi đó:

*a) $\cos x > 0$

*c) $\sin x = \frac{\sqrt{6}}{3}$

*b) $\cos x = \frac{\sqrt{3}}{3}$

d) $\cos(x - 30^\circ) = \frac{3 - \sqrt{6}}{6}$

Lời giải

a) Đúng	b) Đúng	c) Đúng	d) Sai
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Vì $0 < x < 90^\circ$ nên $\cos x > 0$.

Ta có: $\frac{1}{\cos^2 x} = 1 + \tan^2 x = 1 + 2 = 3 \Rightarrow \cos^2 x = \frac{1}{3} \Rightarrow \cos x = \frac{\sqrt{3}}{3}$

Mặt khác: $\tan x = \frac{\sin x}{\cos x} \Rightarrow \sin x = \tan x \cos x = \frac{\sqrt{6}}{3}$.

$$\cos(x - 30^\circ) = \cos x \cos 30^\circ + \sin x \sin 30^\circ = \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{6}}{3} \cdot \frac{1}{2} = \frac{3 + \sqrt{6}}{6}$$

Câu 8. Biết $\sin a = \frac{8}{17}, \tan b = \frac{5}{12}$ và a, b là các góc nhọn. Khi đó:

*a) $\tan a = \frac{8}{15}$

*b) $\sin(a - b) = \frac{21}{221}$

c) $\cos(a + b) = \frac{14}{22}$

d) $\tan(a + b) = \frac{17}{14}$.

Lời giải

a) Đúng	b) Đúng	c) Sai	d) Sai
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Vì a, b là các góc nhọn nên $\cos a > 0, \cos b > 0$.

Ta có: $\cos a = \sqrt{1 - \sin^2 a} = \frac{15}{17} \Rightarrow \tan a = \frac{\sin a}{\cos a} = \frac{8}{15}$;

$$\cos b = \frac{1}{\sqrt{1 + \tan^2 b}} = \frac{12}{13} \Rightarrow \sin b = \cos b \tan b = \frac{5}{13}.$$

Khi

đó:

$$\sin(a - b) = \sin a \cos b - \cos a \sin b = \frac{8}{17} \cdot \frac{12}{13} - \frac{15}{17} \cdot \frac{5}{13} = \frac{21}{221}.$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b = \frac{15}{17} \cdot \frac{12}{13} - \frac{8}{17} \cdot \frac{5}{13} = \frac{140}{221}$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} = \frac{\frac{8}{15} + \frac{5}{12}}{1 - \frac{8}{15} \cdot \frac{5}{12}} = \frac{171}{140}.$$

Câu 9. Biết $0 < a, b < \frac{\pi}{2}, a + b = \frac{\pi}{4}$ và $\tan a \tan b = 3 - 2\sqrt{2}$. Khi đó:

*a) $\tan a + \tan b = -2 + 2\sqrt{2}$.

*b) $\tan a = -1 + \sqrt{2}$

c) $\tan b = -1 - \sqrt{2}$

d) $\tan a - \tan b = -2 - 2\sqrt{2}$.

Lời giải

a) Đúng	b) Đúng	c) Sai	d) Sai
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$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \Rightarrow \tan a + \tan b = \tan(a + b)(1 - \tan a \tan b) \quad \text{mà } a + b = \frac{\pi}{4} \text{ và } \tan a \tan b = 3 - 2\sqrt{2}$$

$$\Rightarrow \tan a + \tan b = \tan \frac{\pi}{4} [1 - (3 - 2\sqrt{2})] = -2 + 2\sqrt{2}.$$

Đặt $S = \tan a + \tan b; P = \tan a \tan b$.

Khi đó $\tan a, \tan b$ là nghiệm của phương trình

$$X^2 - SX + P = 0 \Leftrightarrow X^2 - (2\sqrt{2} - 2)X + 3 - 2\sqrt{2} = 0$$

$$\Leftrightarrow X = -1 + \sqrt{2}. \text{ Suy ra } \tan a = \tan b = -1 + \sqrt{2}.$$

Câu 10. Biết $\tan \alpha = 2$. Khi đó:

a) $\cot \alpha = -\frac{1}{2}$

*b) $\cos 2\alpha = -\frac{3}{5}$

*c) $\sin 2\alpha = \frac{4}{5}$

*d) $\tan 2\alpha = -\frac{4}{3}$

Lời giải

a) Sai	b) Đúng	c) Đúng	d) Đúng
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Ta có:

$$\cos 2\alpha = 2\cos^2 \alpha - 1 = \frac{2}{\tan^2 \alpha + 1} - 1 = \frac{2}{5} - 1 = -\frac{3}{5};$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha = 2\tan \alpha \cos^2 \alpha = \frac{2\tan \alpha}{\tan^2 \alpha + 1} = \frac{4}{5}; \tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = -\frac{4}{3}$$

Câu 11. Biết $\sin 2\alpha = -\frac{4}{5}, \frac{\pi}{2} < \alpha < \frac{3\pi}{2}$. Khi đó:

*a) $\cos \alpha < 0$

*b) $2\sin \alpha \cos \alpha = -\frac{4}{5}$

*c) $\cos \alpha = \frac{-2}{\sqrt{5}}, \sin \alpha = \frac{1}{\sqrt{5}}$

d) $\cos \alpha = \frac{-1}{\sqrt{5}}, \sin \alpha = -\frac{2}{\sqrt{5}}$

Lời giải

a) Đúng	b) Đúng	c) Đúng	d) Sai
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Vì $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ nên $\cos \alpha < 0$. Ta có hệ:

$$\begin{cases} \sin^2 \alpha + \cos^2 \alpha = 1 \\ 2\sin \alpha \cos \alpha = -\frac{4}{5} \end{cases}$$

$$\Rightarrow \begin{cases} \frac{4}{25\cos^2 \alpha} + \cos^2 \alpha = 1 \\ \sin \alpha = -\frac{2}{5\cos \alpha} \end{cases} \Rightarrow \begin{cases} 25\cos^4 \alpha - 25\cos^2 \alpha + 4 = 0 \\ \sin \alpha = -\frac{2}{5\cos \alpha} \end{cases}$$

$$\Rightarrow \begin{cases} \cos^2 \alpha = \frac{4}{5} \\ \cos^2 \alpha = \frac{1}{5} \\ \sin \alpha = -\frac{2}{5\cos \alpha} \end{cases} \Rightarrow \begin{cases} \cos \alpha = \frac{-2}{\sqrt{5}} \\ \cos \alpha = \frac{-1}{\sqrt{5}} \\ \sin \alpha = -\frac{2}{5\cos \alpha} \end{cases} \Rightarrow \begin{cases} \cos \alpha = \frac{-2}{\sqrt{5}}, \sin \alpha = \frac{1}{\sqrt{5}} \\ \cos \alpha = \frac{-1}{\sqrt{5}}, \sin \alpha = \frac{2}{\sqrt{5}} \end{cases}$$

Câu 12. Cho $\sin \alpha = \frac{2}{3}, \frac{\pi}{2} < \alpha < \pi$. Khi đó:

*a) $\cos \alpha = -\frac{\sqrt{5}}{3}$

*b) $\tan \alpha = -\frac{2\sqrt{5}}{5}$

c) $\cos\left(\frac{\pi}{3} + \alpha\right) = \frac{\sqrt{5} - 2\sqrt{3}}{6}$

d) $\cos\left(\frac{\pi}{4} - \alpha\right) = \frac{\sqrt{10} - 2\sqrt{2}}{6}$

Lời giải

a) Đúng	b) Đúng	c) Sai	d) Sai
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$$\sin \alpha = \frac{2}{3}, \frac{\pi}{2} < \alpha < \pi$$

Ta có:

$$\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9} \Rightarrow \cos \alpha = \pm \frac{\sqrt{5}}{3}$$

$$\text{Vì } \frac{\pi}{2} < \alpha < \pi \quad \text{nên} \quad \cos \alpha = -\frac{\sqrt{5}}{3}$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = -\frac{2\sqrt{5}}{5}$$

$$\cos\left(\frac{\pi}{3} + \alpha\right) = \cos \frac{\pi}{3} \cos \alpha - \sin \frac{\pi}{3} \sin \alpha = \frac{1}{2} \left(-\frac{\sqrt{5}}{3}\right) - \frac{\sqrt{3}}{2} \cdot \frac{2}{3} = -\frac{\sqrt{5} + 2\sqrt{3}}{6}$$

$$\cos\left(\frac{\pi}{4} - \alpha\right) = \cos \frac{\pi}{4} \cos \alpha + \sin \frac{\pi}{4} \sin \alpha = \frac{\sqrt{2}}{2} \left(-\frac{\sqrt{5}}{3}\right) + \frac{\sqrt{2}}{2} \cdot \frac{2}{3} = -\frac{\sqrt{10} + 2\sqrt{2}}{6}$$

Câu 13. Cho $\cot x = -\sqrt{3}, \frac{3\pi}{2} < x < 2\pi$. Khi đó:

*a) $\sin x = -\frac{\sqrt{10}}{10}$

b) $\cos x = \frac{\sqrt{3}}{10}$

*c) $\sin\left(\frac{4\pi}{3} - x\right) = \frac{-\sqrt{10}}{5}$

*d) $\tan\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$

Lời giải

a) Đúng	b) Sai	c) Đúng	d) Đúng
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$$\cot x = -\sqrt{3}, \frac{3\pi}{2} < x < 2\pi$$

$$\frac{1}{\sin^2 x} = 1 + \cot^2 x = 1 + (-\sqrt{3})^2 = 10 \Rightarrow \sin^2 x = \frac{1}{10} \Rightarrow \sin x = \pm \frac{\sqrt{10}}{10}$$

$$\text{Vì } \frac{3\pi}{2} < x < 2\pi \text{ nên } \sin x = -\frac{\sqrt{10}}{10}$$

$$\cos x = \cot x \cdot \sin x = -\sqrt{3} \cdot \left(-\frac{\sqrt{10}}{10}\right) = \frac{\sqrt{30}}{10}$$

$$\sin\left(\frac{4\pi}{3} - x\right) = \sin \frac{4\pi}{3} \cos x - \cos \frac{4\pi}{3} \sin x = -\frac{\sqrt{3}}{2} \cdot \left(\frac{\sqrt{30}}{10}\right) - \frac{-1}{2} \cdot \frac{\sqrt{10}}{10} = -\frac{\sqrt{10}}{5}$$

$$\tan x = \frac{1}{\cot x} = -\frac{\sqrt{3}}{3}$$

$$\tan\left(x + \frac{\pi}{3}\right) = \frac{\tan x + \tan \frac{\pi}{3}}{1 - \tan x \cdot \tan \frac{\pi}{3}} = \frac{-\frac{\sqrt{3}}{3} + \sqrt{3}}{1 - \frac{-\sqrt{3}}{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$$

Câu 14. Biết: $\sin \alpha = \frac{1}{3}$ và $0 < \alpha < \frac{\pi}{2}$. Khi đó:

a) $\sin 2\alpha = \frac{\sqrt{2}}{9}$

*b) $\cos 2\alpha = \frac{7}{9}$

c) $\tan 2\alpha = \frac{\sqrt{2}}{7}$

d) $\cot 2\alpha = \frac{7\sqrt{2}}{2}$

Lời giải

a) Sai	b) Đúng	c) Sai	d) Sai
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Ta có: $\sin \alpha = \frac{1}{3}$ và $0 < \alpha < \frac{\pi}{2}$.

$$\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \left(\frac{1}{3}\right)^2 = \frac{8}{9} \Rightarrow \cos \alpha = \pm \frac{2\sqrt{2}}{3}$$

Vì $0 < \alpha < \frac{\pi}{2}$ nên $\cos \alpha = \frac{2\sqrt{2}}{3}$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha = 2 \cdot \frac{1}{3} \cdot \frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{9}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \left(\frac{2\sqrt{2}}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{7}{9}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{4\sqrt{2}}{2} : \frac{7}{9} = \frac{4\sqrt{2}}{7}$$

$$\cot 2\alpha = \frac{\cos 2\alpha}{\sin 2\alpha} = \frac{4\sqrt{2}}{2} : \frac{7}{9} = \frac{7\sqrt{2}}{8}.$$

Câu 15. Cho $\cos \alpha = \frac{2}{5}, 2\pi < \alpha < \frac{5\pi}{2}$, khi đó:

*a) $\sin \alpha = \frac{\sqrt{21}}{5}$

*b) $\sin 2\alpha = \frac{4\sqrt{21}}{25}$

*c) $\cos 2\alpha = -\frac{17}{25}$

*d) $\tan 2\alpha = \frac{-4\sqrt{21}}{17}$

Lời giải

a) Đúng	b) Đúng	c) Đúng	d) Đúng
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$$\cos \alpha = \frac{2}{5}, 2\pi < \alpha < \frac{5\pi}{2}.$$

$$\sin^2 \alpha = 1 - \cos^2 \alpha = 1 - \left(\frac{2}{5}\right)^2 = \frac{21}{25} \Rightarrow \sin \alpha = \pm \frac{\sqrt{21}}{5}$$

Vì $2\pi < \alpha < \frac{5\pi}{2}$ nên $\sin \alpha = \frac{\sqrt{21}}{5}$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha = 2 \cdot \frac{\sqrt{21}}{5} \cdot \frac{2}{5} = \frac{4\sqrt{21}}{25}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \left(\frac{2}{5}\right)^2 - \left(\frac{\sqrt{21}}{5}\right)^2 = -\frac{17}{25}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{4\sqrt{21}}{25} : \frac{-17}{25} = \frac{-4\sqrt{21}}{17}.$$

Câu 16. Biết: $\cos 2\alpha = \frac{5}{9}, 0^\circ < \alpha < 90^\circ$. Khi đó:

*a) $\sin \alpha = \frac{\sqrt{28}}{9}$

*b) $\cos \alpha = \frac{\sqrt{53}}{9}$

c) $\tan \alpha = \frac{\sqrt{371}}{53}$

*d) $\cot \alpha = \frac{\sqrt{371}}{14}$

Lời giải

a) Đúng	b) Đúng	c) Sai	d) Đúng
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Ta có: $\cos 2\alpha = \frac{5}{9}, 0^\circ < \alpha < 90^\circ$

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2} = \frac{1 - \left(\frac{5}{9}\right)^2}{2} = \frac{28}{81} \Rightarrow \sin \alpha = \pm \frac{\sqrt{28}}{9}$$

Vì $0^\circ < \alpha < 90^\circ$ nên $\sin \alpha = \frac{\sqrt{28}}{9}$

$$\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \left(\frac{\sqrt{28}}{9}\right)^2 = \frac{53}{81} \Rightarrow \cos \alpha = \pm \frac{\sqrt{53}}{9}$$

Vì $0^\circ < \alpha < 90^\circ$ nên $\cos \alpha = \frac{\sqrt{53}}{9}$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{2\sqrt{371}}{53}$$

$$\cot \alpha = \frac{1}{\tan \alpha} = \frac{\sqrt{371}}{14}.$$

Câu 17. Biến đổi được các biểu thức sau về dạng tích số. Khi đó:

- a) $\cos 3x + \cos x = 2 \cos 2x \cdot \cos 3x$ b) $\sin 3x + \sin 2x = 2 \sin 2x \cos \frac{x}{2};$
- *c) $\cos 4x - \cos x = -2 \sin \frac{5x}{2} \sin \frac{3x}{2}$ *d) $\sin 5x - \sin x = 2 \cos 3x \sin 2x$

Lời giải

a) Sai	b) Sai	c) Đúng	d) Đúng
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a) $\cos 3x + \cos x = 2 \cos 2x \cdot \cos x$

b) $\sin 3x + \sin 2x = 2 \sin \frac{5x}{2} \cos \frac{x}{2};$

c) $\cos 4x - \cos x = -2 \sin \frac{5x}{2} \sin \frac{3x}{2}$

d) $\sin 5x - \sin x = 2 \cos 3x \sin 2x$

Câu 18. Cho $\sin x = \frac{1}{5}, \frac{\pi}{2} < x < \pi$. Khi đó:

- a) $\sin 2x = \frac{4\sqrt{6}}{5}.$ *b) $\cos 2x = \frac{23}{25}$ c) $\tan 2x = \frac{20\sqrt{6}}{3}$ d) $\cot 2x = \frac{23\sqrt{6}}{120}$

Lời giải

a) Sai	b) Đúng	c) Sai	d) Sai
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$$\frac{\pi}{2} < x < \pi \Rightarrow \frac{2\pi}{2} < 2x < 2\pi \Rightarrow \pi < 2x < 2\pi \Rightarrow \sin 2x < 0.$$

$$\frac{\pi}{2} < x < \pi \Rightarrow \cos x < 0$$

$$\sin x = \frac{1}{5} \Rightarrow \cos x = -\sqrt{1 - \sin^2 x} = -\frac{2\sqrt{6}}{5}.$$

$$\sin 2x = 2 \sin x \cdot \cos x = 2 \cdot \frac{1}{5} \cdot \left(-\frac{2\sqrt{6}}{5}\right) = -\frac{4\sqrt{6}}{5}.$$

$$\frac{\pi}{2} < x < \pi \Rightarrow \frac{2\pi}{2} < 2x < 2\pi \Rightarrow \pi < 2x < 2\pi \Rightarrow \cos 2x > 0.$$

$$\cos 2x = 1 - 2 \sin^2 x = 1 - 2 \cdot \frac{1}{25} = \frac{23}{25}.$$

$$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{-\frac{4\sqrt{6}}{5}}{\frac{23}{25}} = -\frac{20\sqrt{6}}{3}, \cot 2x = \frac{\cos 2x}{\sin 2x} = \frac{\frac{23}{25}}{-\frac{4\sqrt{6}}{5}} = -\frac{23\sqrt{6}}{120}.$$

Câu 19. Cho $\cos x = \frac{1}{5}, \frac{\pi}{2} < x < \pi$. Khi đó:

a) $\sin \frac{x}{2} = \frac{\sqrt{10}}{4}$

b) $\cos \frac{x}{2} = \frac{\sqrt{15}}{4}$

*c) $\tan \frac{x}{2} = \frac{\sqrt{6}}{3}$

*d) $\cot \frac{x}{2} = \frac{\sqrt{6}}{2}$

Lời giải

a) Sai	b) Sai	c) Đúng	d) Đúng
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$$\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2} \Rightarrow \sin \frac{x}{2} > 0.$$

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 - \frac{1}{5}}{2}} = \frac{\sqrt{10}}{5}.$$

$$\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2} \Rightarrow \cos \frac{x}{2} > 0.$$

$$\cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}} = \sqrt{\frac{1 + \frac{1}{5}}{2}} = \frac{\sqrt{15}}{5}.$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\frac{\sqrt{10}}{5}}{\frac{\sqrt{15}}{5}} = \frac{\sqrt{6}}{3}.$$

$$\cot \frac{x}{2} = \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} = \frac{\frac{\sqrt{15}}{5}}{\frac{\sqrt{10}}{5}} = \frac{\sqrt{6}}{2}.$$

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