CD1: THỰC HIỆN PHÉP TÍNH

DẠNG 9: TÍNH TỈ SỐ CỦA HAI TỔNG

A. Trắc nghiệm

Câu 1. **(HSG 7 huyện Yên Thế - Bắc Giang 2022 - 2023)**

$$A=\frac{2-\frac{2}{19}+\frac{2}{43}-\frac{2}{2023}}{3-\frac{3}{19}+\frac{3}{43}-\frac{3}{2023}}$$

Kết quả phép tính là:

A. $\frac{2}{3}$. **B.** $\frac{4}{5}$. **C.** $\frac{2}{5}$. **D.** $\frac{5}{6}$.

Lời giải

Chọn A

$$A=\frac{2-\frac{2}{19}+\frac{2}{43}-\frac{2}{2023}}{3-\frac{3}{19}+\frac{3}{43}-\frac{3}{2023}}=\frac{2\cdot\left(1-\frac{1}{19}+\frac{1}{43}-\frac{1}{2023}\right)}{3\cdot\left(1-\frac{1}{19}+\frac{1}{43}-\frac{1}{2023}\right)}=\frac{2}{3}$$

Câu 2. **(HSG 7 huyện Tân Yên - Bắc Giang 2022 - 2023)**

$$A=\frac{2-\frac{2}{19}+\frac{2}{43}-\frac{2}{1943}}{3-\frac{3}{19}+\frac{3}{43}-\frac{3}{1943}}$$

Kết quả phép tính là:

A. $\frac{2}{3}$. **B.** $\frac{4}{5}$. **C.** $\frac{2}{5}$. **D.** $\frac{5}{6}$.

Lời giải

Chọn A

$$A=\frac{2-\frac{2}{19}+\frac{2}{43}-\frac{2}{1943}}{3-\frac{3}{19}+\frac{3}{43}-\frac{3}{1943}}=\frac{2\cdot\left(1-\frac{1}{19}+\frac{1}{43}-\frac{1}{1943}\right)}{3\cdot\left(1-\frac{1}{19}+\frac{1}{43}-\frac{1}{1943}\right)}=\frac{2}{3}$$

Câu 3. **(HSG 7 huyện Thanh Sơn, tỉnh Phú Thọ, 2021 - 2022)**

$$\frac{\frac{1}{9}-\frac{1}{7}-\frac{1}{11}}{\frac{4}{9}-\frac{4}{7}-\frac{4}{11}}+\frac{0,6-\frac{3}{25}-\frac{3}{125}-\frac{3}{625}}{\frac{4}{5}-0,16-\frac{4}{125}-\frac{4}{625}}$$

Giá trị biểu thức là:

A. 0. **B.** 1. **C.** 2. **D.** 3.

Lời giải

Chọn B

$$\frac{\frac{1}{9}-\frac{1}{7}-\frac{1}{11}}{\frac{4}{9}-\frac{4}{7}-\frac{4}{11}}+\frac{0,6-\frac{3}{25}-\frac{3}{125}-\frac{3}{625}}{\frac{4}{5}-0,16-\frac{4}{125}-\frac{4}{625}}$$

$$=\frac{\frac{1}{9}-\frac{1}{7}-\frac{1}{11}}{4\left(\frac{1}{9}-\frac{1}{7}-\frac{1}{11}\right)}+\frac{\frac{3}{5}-\frac{3}{25}-\frac{3}{125}-\frac{3}{625}}{\frac{4}{5}-\frac{4}{25}-\frac{4}{125}-\frac{4}{625}}$$

$$=\frac{\frac{1}{9}-\frac{1}{7}-\frac{1}{11}}{4\left(\frac{1}{9}-\frac{1}{7}-\frac{1}{11}\right)}+\frac{3\left(\frac{1}{5}-\frac{1}{25}-\frac{1}{125}-\frac{1}{625}\right)}{4\left(\frac{1}{5}-\frac{1}{25}-\frac{1}{125}-\frac{1}{625}\right)}$$

$$=\frac{1}{4}+\frac{3}{4}=\frac{4}{4}=1$$

B. Tự luận

Câu 1. **(HSG 7 Lai Châu năm 2022-2023)**

Tính giá trị biểu thức:

$$A=5.\left[\frac{12+\frac{12}{289}-\frac{12}{7}-\frac{12}{85}}{4-\frac{4}{7}-\frac{4}{85}+\frac{4}{289}}:\frac{5+\frac{5}{13}+\frac{5}{95}+\frac{5}{169}}{6+\frac{6}{13}+\frac{6}{169}+\frac{6}{95}}\right].\frac{166166166}{747747747}$$

Lời giải

$$A=5\cdot\left[\frac{12+\frac{12}{289}-\frac{12}{7}-\frac{12}{85}}{4-\frac{4}{7}-\frac{4}{85}+\frac{4}{289}}:\frac{5+\frac{5}{13}+\frac{5}{95}+\frac{5}{169}}{6+\frac{6}{13}+\frac{6}{169}+\frac{6}{95}}\right].\frac{166166166}{747747747}$$

$$A=5.\left[\frac{12\left(1+\frac{1}{289}-\frac{1}{7}-\frac{1}{85}\right)}{4\left(1-\frac{1}{7}-\frac{1}{85}+\frac{1}{289}\right)}:\frac{5\left(1+\frac{1}{13}+\frac{1}{95}+\frac{1}{169}\right)}{6\left(1+\frac{1}{13}+\frac{1}{169}+\frac{1}{95}\right)}\right].\frac{2.83083083}{9.83083083}$$

$$A=5.\left[\frac{12\left(1+\frac{1}{289}-\frac{1}{7}-\frac{1}{85}\right)}{4\left(1+\frac{1}{289}-\frac{1}{7}-\frac{1}{85}\right)}:\frac{5\left(1+\frac{1}{13}+\frac{1}{95}+\frac{1}{169}\right)}{6\left(1+\frac{1}{13}+\frac{1}{169}+\frac{1}{95}\right)}\right].\frac{2.83083083}{9.83083083}$$

$$A=5.\left[\frac{12}{4}:\frac{5}{6}\right].\frac{2}{9}=5.\left[3.\frac{6}{5}\right].\frac{2}{9}=5.\frac{18}{5}.\frac{2}{9}=4$$

Vậy $A=4$

Câu 2. **(HSG 7 Thọ Xuân Thanh Hoá năm 2022-2023)**

$$A=\frac{\frac{3}{4}-\frac{3}{11}+\frac{3}{13}}{\frac{5}{4}-\frac{5}{11}+\frac{5}{13}}-\frac{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}{\frac{5}{2}-\frac{5}{3}+\frac{5}{4}}$$

Tính giá trị biểu thức:

Lời giải

$$A=\frac{\frac{3}{4}-\frac{3}{11}+\frac{3}{13}}{\frac{5}{4}-\frac{5}{11}+\frac{5}{13}}-\frac{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}{\frac{5}{2}-\frac{5}{3}+\frac{5}{4}}=\frac{3.\left(\frac{1}{4}-\frac{1}{11}+\frac{1}{13}\right)}{5.\left(\frac{1}{4}-\frac{1}{11}+\frac{1}{13}\right)}-\frac{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}{5.\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{4}\right)}=\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$$

Câu 3. **(HSG 7 Diễm Châu năm 2022-2023)**

Tính giá trị biểu thức: $\left(\frac{0,4-\frac{2}{9}+\frac{2}{11}}{1,4-\frac{7}{9}+\frac{7}{11}}-\frac{\frac{1}{3}-0,25+\frac{1}{5}}{1\frac{1}{6}-0,875+0,7}\right):\frac{2022}{2023}$

Lời giải

$$\left(\frac{0,4-\frac{2}{9}+\frac{2}{11}}{1,4-\frac{7}{9}+\frac{7}{11}}-\frac{\frac{1}{3}-0,25+\frac{1}{5}}{1\frac{1}{6}-0,875+0,7}\right):\frac{2022}{2023}$$

$$=\left(\frac{2\left(\frac{1}{5}-\frac{1}{9}+\frac{1}{11}\right)}{7\left(\frac{1}{5}-\frac{1}{9}+\frac{1}{11}\right)}-\frac{\left(\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\right)}{\frac{7}{2}\left(\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\right)}\right):\frac{2022}{2023}$$

$$=\left(\frac{2}{7}-\frac{2}{7}\right):\frac{2022}{2023}=0$$

Câu 4. **(HSG Hưng Hà 2022-2023)**

$$A=\left(\frac{0,375-0,3+\frac{3}{11}+\frac{3}{12}}{-0,625+0,5-\frac{5}{11}-\frac{5}{12}}-\frac{1+\frac{2}{17}-\frac{2}{19}}{2,5+\frac{5}{17}-\frac{5}{19}}\right).1\frac{3}{2020}$$

Thực hiện phép tính:

Lời giải

$$A=\left(\frac{0,375-0,3+\frac{3}{11}+\frac{3}{12}}{-0,625+0,5-\frac{5}{11}-\frac{5}{12}}-\frac{1+\frac{2}{17}-\frac{2}{19}}{2,5+\frac{5}{17}-\frac{5}{19}}\right).1\frac{3}{2020}$$

$$=\left[\frac{3.\left(0,125-0,1+\frac{1}{11}+\frac{1}{12}\right)}{-5.(0,125-0,1+\frac{1}{11}+\frac{1}{12}}-\frac{2\left(0,5+\frac{1}{17}-\frac{1}{19}\right)}{5\left(0,5+\frac{1}{17}-\frac{1}{19}\right)}\right].\frac{2023}{2020}$$

$$=\left(\frac{-3}{5}-\frac{2}{5}\right).\frac{2023}{2020}=-1.\frac{2023}{2020}=-\frac{2023}{2020}$$

Câu 5. **(HSG 7 Nghi Sơn năm 2022-2023)**

Thực hiện phép tính: $B=\frac{\frac{1}{9}-\frac{1}{7}-\frac{1}{11}}{\frac{4}{9}-\frac{4}{7}-\frac{4}{11}}+\frac{0,6-\frac{3}{25}-\frac{3}{125}-\frac{3}{625}}{\frac{4}{5}-0,16-\frac{4}{125}-\frac{4}{625}}$

Lời giải

$$B=\frac{\frac{1}{9}-\frac{1}{7}-\frac{1}{11}}{\frac{4}{9}-\frac{4}{7}-\frac{4}{11}}+\frac{0,6-\frac{3}{25}-\frac{3}{125}-\frac{3}{625}}{\frac{4}{5}-0,16-\frac{4}{125}-\frac{4}{625}}$$

$$=\frac{\frac{1}{9}-\frac{1}{7}-\frac{1}{11}}{4.\left(\frac{1}{9}-\frac{1}{7}-\frac{1}{11}\right)}+\frac{3.\left(0,2-\frac{1}{25}-\frac{1}{125}-\frac{1}{625}\right)}{4.\left(0,2-\frac{1}{25}-\frac{1}{125}-\frac{1}{625}\right)}=\frac{1}{4}+\frac{3}{4}=1$$

Câu 6. **(HSG 7 Đông Hưng năm 2022-2023)**

$$A=\frac{2022}{2023}:\left(\frac{0,4-\frac{2}{11}+\frac{2}{13}}{1,4-\frac{7}{11}+\frac{7}{13}}+\frac{2,5-\frac{5}{3}+1,25}{3,5-2\frac{1}{3}+1,75}\right)$$

Thực hiện phép tính sau

Lời giải

$$A=\frac{2022}{2023}:\left(\frac{0,4-\frac{2}{11}+\frac{2}{13}}{1,4-\frac{7}{11}+\frac{7}{13}}+\frac{2,5-\frac{5}{3}+1,25}{3,5-2\frac{1}{3}+1,75}\right)$$

$$=\frac{2022}{2023}:\left[\frac{2\cdot\left(\frac{1}{5}-\frac{1}{11}+\frac{1}{13}\right)}{7\cdot\left(\frac{1}{5}-\frac{1}{11}+\frac{1}{13}\right)}+\frac{5\cdot\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{4}\right)}{7\cdot\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{4}\right)}\right.$$

$$=\frac{2022}{2023}:\left(\frac{2}{7}+\frac{5}{7}\right)=\frac{2022}{2023}$$

Câu 7. **(HSG 7 Hà Trung năm 2022-2023)**

$$C=\frac{\frac{3}{4}-\frac{3}{11}+\frac{3}{13}}{\frac{5}{4}-\frac{5}{11}+\frac{5}{13}}+\frac{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}{\frac{5}{4}-\frac{5}{6}+\frac{5}{8}}$$

Tính giá trị các biểu thức sau

Lời giải

$$C=\frac{\frac{3}{4}-\frac{3}{11}+\frac{3}{13}}{\frac{5}{4}-\frac{5}{11}+\frac{5}{13}}+\frac{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}{\frac{5}{4}-\frac{5}{6}+\frac{5}{8}}=\frac{3\left(\frac{1}{4}-\frac{1}{11}+\frac{1}{13}\right)}{5\left(\frac{1}{4}-\frac{1}{11}+\frac{1}{13}\right)}+\frac{2\left(\frac{1}{4}-\frac{1}{6}+\frac{1}{8}\right)}{5\left(\frac{1}{4}-\frac{1}{6}+\frac{1}{8}\right)}=\frac{3}{5}+\frac{2}{5}=1$$

Câu 8. **(HSG 7 Lập Thạch-Thanh Hoá năm 2022-2023)**

$$A=\frac{\frac{1}{3}-\frac{1}{7}-\frac{1}{13}}{\frac{2}{3}-\frac{2}{7}-\frac{2}{13}}\cdot\frac{\frac{3}{4}-\frac{3}{16}-\frac{3}{64}-\frac{3}{256}}{1-\frac{1}{4}-\frac{1}{16}-\frac{1}{64}}+\frac{5}{8}$$

Tính bằng cách hợp lí:

Lời giải

$$A=\frac{\frac{1}{3}-\frac{1}{7}-\frac{1}{13}}{2\left(\frac{1}{3}-\frac{1}{7}-\frac{1}{13}\right)}\cdot\frac{\frac{3}{4}\left(1-\frac{1}{4}-\frac{1}{16}-\frac{1}{64}\right)}{1-\frac{1}{4}-\frac{1}{16}-\frac{1}{64}}+\frac{5}{8}=\frac{1}{2}\cdot\frac{3}{4}+\frac{5}{8}=1$$

Vậy $A=1$

Câu 7. **(HSG 7 Thi thử huyện Thanh Miện 2022 - 2023)**

Thực hiện phép tính: $P=\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2022}}{\frac{2021}{1}+\frac{2020}{2}+\frac{2019}{3}+...+\frac{2}{2020}+\frac{1}{2021}}$

Lời giải

$$P=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2022}}{\frac{2021}{1}+\frac{2020}{2}+\frac{2019}{3}+...+\frac{2}{2020}+\frac{1}{2021}}$$

$$=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2022}}{\frac{2022-1}{1}+\frac{2022-2}{2}+\frac{2022-3}{3}+...+\frac{2022-2020}{2020}+\frac{2022-2021}{2021}}$$

$$=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2022}}{\frac{2022}{1}+\frac{2022}{2}+\frac{2022}{3}+...+\frac{2022}{2020}+\frac{2022}{2021}-\left(\frac{1}{1}+\frac{2}{2}+\frac{3}{3}+...+\frac{2020}{2020}+\frac{2021}{2021}\right)}$$

$$=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2022}}{2022.\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2020}+\frac{1}{2021}\right)-2021}$$

$$=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2022}}{2022+2022\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2020}+\frac{1}{2021}\right)-2021}$$

$$=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2022}}{1+2022\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2020}+\frac{1}{2021}\right)}$$

$$=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2022}}{2022\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2020}+\frac{1}{2021}+\frac{1}{2022}\right)}=\frac{1}{2022}$$

Câu 9. **(HSG 7 TP Bắc Giang 2022 - 2023)**

Tính tỉ số $\dfrac{A}{B}$ biết $A=92-\frac{1}{9}-\frac{2}{10}-\frac{3}{11}-...-\frac{92}{100}$ và $B=\frac{1}{45}+\frac{1}{50}+\frac{1}{55}+...+\frac{1}{500}$.

Lời giải

$$A=\left(1-\frac{1}{9}\right)+\left(1-\frac{2}{10}\right)+\left(1-\frac{3}{11}\right)+.....+\left(1-\frac{92}{100}\right)$$

$$=\frac{8}{9}+\frac{8}{10}+\frac{8}{11}+.....+\frac{8}{100}$$

$$=8\cdot\left(\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+.....+\frac{1}{100}\right)$$

$$B=\frac{1}{5}\cdot\left(\frac{1}{9}+\frac{1}{10}+.....+\frac{1}{100}\right)$$

Lại có

Thay vào tỉ số $\frac{A}{B}$ ta được

$$\frac{A}{B}=\frac{8\cdot\left(\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+.....+\frac{1}{100}\right)}{\frac{1}{5}\cdot\left(\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+.....+\frac{1}{100}\right)}=\frac{8}{\frac{1}{5}}=40$$

Vậy A=40

Câu 10. **(HSG 7 huyện Hưng Hà, tỉnh, trường Nguyễn Tông Quai 2022 - 2023)**

Cho $|2023x_1-2022y_1|+|2023x_2-2022y_2|+...+|2023x_{100}-2022y_{100}|\le 0$.

Tính $\frac{x_1+x_2+x_3+...+x_{100}}{y_1+y_2+y_3+...+y_{100}}$.

Lời giải

Ta có

$$|2023x_1-2022y_1|\ge 0$$

$$|2023x_2-2022y_2|\ge 0$$

…

$$|2023x_{100}-2022y_{100}|\ge 0$$

$$\Rightarrow (2023x_1-2022y_1)^2+(2023x_2-2022y_2)^2+...+(2023x_{100}-2022y_{100})^2\ge 0$$

Theo bài ra ta có: $|2023x_1-2022y_1|+|2023x_2-2022y_2|+...+|2023x_{100}-2022y_{100}|\le 0$

$$\Rightarrow\begin{cases}|2023x_1-2022y_1|=0\\|2023x_2-2022y_2|=0\\\vdots\\|2023x_{100}-2022y_{100}|=0\end{cases}$$

$$\Rightarrow\begin{cases}2023x_1=2022y_1\\2023x_2=2022y_2\\\vdots\\2023x_{100}=2022y_{100}\end{cases}\Rightarrow\frac{x_1}{y_1}=\frac{x_2}{y_2}=...=\frac{x_{100}}{y_{100}}=\frac{2022}{2023}\qquad(1)$$

Áp dụng tính chất của dãy tỉ số bằng nhau ta được:

$$\frac{x_1}{y_1}=\frac{x_2}{y_2}=...=\frac{x_{100}}{y_{100}}=\frac{x_1+x_2+...+x_{100}}{y_1+y_2+...+y_{100}} \quad (2)$$

Từ (1) và (2) suy ra $\frac{x_1+x_2+x_3+...+x_{100}}{y_1+y_2+y_3+...+y_{100}}=\frac{2022}{2023}$ (đpcm)

Câu 11. **(HSG 7 huyện Hưng Hà, tỉnh, trường Bắc Sơn 2022 - 2023)**

Tính hợp lí giá trị biểu thức: $B=\frac{2021.1+2020.2+2019.3+...+2.2020+1.2021}{1.2+2.3+3.4+...+2020.2021+2021.2022}$

Lời giải

$$B=\frac{2021.1+2020.2+2019.3+...+2.2020+1.2021}{1.2+2.3+3.4+...+2020.2021+2021.2022}$$

Đặt $C=2021.1+2020.2+2019.3+...+2.2020+1.2021$

$$=(1+2+...+2021)+(1+2+...+2020)+...+(1+2)+1$$

$$=\frac{2021.2022}{2}+\frac{2020.2021}{2}+...+\frac{2.3}{2}+\frac{1.2}{2}$$

$$=\frac{1}{2}.(1.2+2.3+...+2020.2021+2021.2022)$$

Do đó $B=\frac{1}{2}$

Câu 12. **(HSG 7 huyện Hưng Hà, tỉnh, trường Trần Thủ Độ 2022 - 2023)**

Tính giá trị của $A=\frac{92-\frac{1}{9}-\frac{2}{10}-\frac{3}{11}-.....-\frac{92}{100}}{\frac{1}{45}+\frac{1}{50}+\frac{1}{55}+.....+\frac{1}{500}}$

Lời giải

$$A=\frac{92-\frac{1}{9}-\frac{2}{10}-\frac{3}{11}-.....-\frac{92}{100}}{\frac{1}{45}+\frac{1}{50}+\frac{1}{55}+.....+\frac{1}{500}}$$

Ta có

$$A=\frac{\left(1-\frac{1}{9}\right)+\left(1-\frac{2}{10}\right)+\left(1-\frac{3}{11}\right)+.....+\left(1-\frac{92}{100}\right)}{\frac{1}{45}+\frac{1}{50}+\frac{1}{55}+.....+\frac{1}{500}}$$

$$A=\frac{\frac{8}{9}+\frac{8}{10}+\frac{8}{11}+.....+\frac{8}{100}}{\frac{1}{45}+\frac{1}{50}+\frac{1}{55}+.....+\frac{1}{500}}$$

$$A=\frac{8.\left(\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+.....+\frac{1}{100}\right)}{\frac{1}{5}.\left(\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+.....+\frac{1}{100}\right)}=40$$

$$A=40$$

Vậy

Câu 13. **(HSG 7 huyện Bá Thước, tỉnh Thanh Hóa, 2021 - 2022)**

Thực hiện phép tính: $D=\frac{2.2022}{1+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+\cdots+\frac{1}{1+2+3+\cdots+2022}}$

Lời giải

$$D=\frac{2.2022}{1+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+\cdots+\frac{1}{1+2+3+\cdots+2022}}$$

$$D=\frac{2.2022}{1+2\left[\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{2022}-\frac{1}{2023}\right)\right]}=\frac{2.2022}{1+2\left(\frac{1}{2}-\frac{1}{2023}\right)}$$

$$=\frac{2.2022.2023}{2.2022}=2023$$

Câu 14. **(Bổ sung)**

Tính các giá trị biểu thức sau: $A=\frac{\frac{1}{2}+\frac{1}{12}+\frac{1}{30}+\cdots+\frac{1}{9120}+\frac{1}{9506}+\frac{1}{9900}}{50-\frac{50}{51}-\frac{51}{52}-\frac{52}{53}-.....-\frac{97}{98}-\frac{98}{99}-\frac{99}{100}}$

Lời giải

Xét tử: $T=\frac{1}{2}+\frac{1}{12}+\frac{1}{30}+\cdots+\frac{1}{9120}+\frac{1}{9506}+\frac{1}{9900}$

$$T=\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\cdots+\frac{1}{95.96}+\frac{1}{97.98}+\frac{1}{99.100}$$

$$T=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....+\frac{1}{97}-\frac{1}{98}+\frac{1}{99}-\frac{1}{100}$$

$$T=\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{97}+\frac{1}{98}+\frac{1}{99}+\frac{1}{100}\right)-2.\left(\frac{1}{2}+\frac{1}{4}+....+\frac{1}{98}+\frac{1}{100}\right)$$

$$T=\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{99}+\frac{1}{100}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{49}+\frac{1}{50}\right)$$

$$T=\frac{1}{51}+\frac{1}{52}+\frac{1}{53}+.....+\frac{1}{99}+\frac{1}{100}\qquad(1)$$

Xét mẫu:

$$M = 50 - \frac{50}{51} - \frac{51}{52} - \frac{52}{53} - \ldots\ldots - \frac{97}{98} - \frac{98}{99} - \frac{99}{100}$$

$$M = \left(1 - \frac{50}{51}\right) + \left(1 - \frac{51}{52}\right) + \ldots\ldots + \left(1 - \frac{98}{99}\right) + \left(1 - \frac{99}{100}\right)$$

$$M = \frac{1}{51} + \frac{1}{52} + \ldots + \frac{1}{99} + \frac{1}{100} \qquad (2)$$

Từ (1) và (2) suy ra: tử bằng mẫu. Do đó: $A = 1$

Câu 15. **(Bổ sung)**

Tính tỉ số $\frac{A}{B}$ biết $A = \frac{2012}{51} + \frac{2012}{52} + \frac{2012}{53} + \ldots + \frac{2012}{100}$ và $B = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots + \frac{1}{99.100}$

Lời giải

Ta có : $A = 2012\left(\frac{1}{51} + \frac{1}{52} + \frac{1}{53} + \ldots + \frac{1}{100}\right)$

$$B = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{99} - \frac{1}{100} = \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots. + \frac{1}{99} + \frac{1}{100}\right) - 2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots + \frac{1}{100}\right)$$

$$B = \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{100}\right) - \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{50}\right) = \frac{1}{51} + \frac{1}{52} + \frac{1}{53} + \ldots + \frac{1}{100}$$

Khi đó: $\frac{A}{B} = \frac{2012}{1} = 2012$

Câu 16. **(Bổ sung)**

Tính tỉ số $\frac{A}{B}$ biết: $A = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots + \frac{1}{101.102}$ và $B = \frac{1}{52.102} + \frac{1}{53.101} + \frac{1}{54.100} + \ldots + \frac{1}{102.52} + \frac{2}{77.154}$

Lời giải

Ta có : $A = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots + \frac{1}{101} - \frac{1}{102} = \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{101} - \frac{1}{102}\right)$

$$A = \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{101} + \frac{1}{102}\right) - 2\left(\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{102}\right)$$

$$A = \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{102}\right) - \left(\frac{1}{1} + \frac{1}{2} + \ldots + \frac{1}{51}\right) = \frac{1}{52} + \frac{1}{53} + \ldots + \frac{1}{101} + \frac{1}{102}$$

$$A = \left(\frac{1}{52} + \frac{1}{102}\right) + \left(\frac{1}{53} + \frac{1}{101}\right) + \ldots + \left(\frac{1}{76} + \frac{1}{78}\right) + \frac{1}{77} = \frac{154}{52.102} + \frac{154}{53.101} + \ldots + \frac{154}{76.78} + \frac{154}{77.154}$$

và $B=\left(\frac{1}{52.102}+\frac{1}{102.52}\right)+\left(\frac{1}{53.101}+\frac{1}{101.53}\right)+...+\left(\frac{1}{76.78}+\frac{1}{78.76}\right)+\frac{2}{77.154}$

$B=\frac{2}{52.102}+\frac{2}{53.101}+...+\frac{2}{76.78}+\frac{2}{77.154}\Rightarrow\frac{A}{B}=\frac{154}{2}=77$

Câu 17. **(Bổ sung)**

Tính tỉ số $\frac{A}{B}$ biết : $A=92-\frac{1}{9}-\frac{2}{10}-\frac{3}{11}-...-\frac{92}{100}$ và $B=\frac{1}{45}+\frac{1}{50}+\frac{1}{55}+...+\frac{1}{500}$

Lời giải

Ta có :

$A=\left(1-\frac{1}{9}\right)+\left(1-\frac{2}{10}\right)+\left(1-\frac{3}{11}\right)+...+\left(1-\frac{92}{100}\right)=\frac{8}{9}+\frac{8}{10}+...+\frac{8}{100}=8\left(\frac{1}{9}+\frac{1}{10}+...+\frac{1}{100}\right)$

$B=\frac{1}{5}\left(\frac{1}{9}+\frac{1}{10}+...+\frac{1}{100}\right)$. Khi đó : $\frac{A}{B}=\frac{8}{\frac{1}{5}}=40$

Câu 18. **(HSG 7 huyện Kinh Môn 2018 - 2019)**

Cho $A=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+....+\frac{1}{2019};B=\frac{1}{2018}+\frac{2}{2017}+\frac{3}{2016}+....+\frac{2017}{2}+\frac{2018}{1}$

Tính $\frac{A}{B}$.

Lời giải

$B=\frac{1}{2018}+\frac{2}{2017}+.....+\frac{2017}{2}+\frac{2018}{1}$

$=\frac{2019-2018}{2018}+\frac{2019-2017}{2017}+.....+\frac{2019-2}{2}+\frac{2019-1}{1}$

$=\frac{2019}{2018}-1+\frac{2019}{2017}-1+.....+\frac{2019}{2}-1+\frac{2019}{1}-1$

$=\frac{2019}{2018}+\frac{2019}{2017}+.....+\frac{2019}{2}+2019-\left(\underbrace{1+1+...+1}_{2018}\right)$

$=2019-2018+\frac{2019}{2018}+\frac{2019}{2017}+.....+\frac{2019}{2}$

$=1+\frac{2019}{2018}+\frac{2019}{2017}+.....+\frac{2019}{2}$

$=2019.\left(\frac{1}{2019}+\frac{1}{2018}+\frac{1}{2017}+.....+\frac{1}{2}\right)$

$=2019.A$

$\Rightarrow\frac{A}{B}=\frac{1}{2019}$

$$\frac{A}{B}=\frac{1}{2019}$$

Vậy

Câu 19. **(HSG 7 trường Ân Tường Đông 2014 - 2015)**

Cho $S=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{48}+\frac{1}{49}+\frac{1}{50}$ và $P=\frac{1}{49}+\frac{2}{48}+\frac{3}{47}+.....+\frac{48}{2}+\frac{49}{1}$

Tính $\frac{S}{P}$.

Lời giải

Ta có: $P=\frac{1}{49}+\frac{2}{48}+\frac{3}{47}+.....+\frac{48}{2}+\frac{49}{1}$

$$=\left(\frac{1}{49}+1\right)+\left(\frac{2}{48}+1\right)+\left(\frac{3}{47}+1\right)+.....+\left(\frac{48}{2}+1\right)+1$$

$$=\left(\frac{50}{49}+\frac{50}{48}+\frac{50}{47}+.....+\frac{50}{2}\right)+1$$

$$=\frac{50}{50}+\frac{50}{49}+\frac{50}{48}+\frac{50}{47}+....\frac{50}{2}$$

$$=50.\left(\frac{1}{50}+\frac{1}{49}+\frac{1}{48}+....+\frac{1}{2}\right)$$

$$\Rightarrow \frac{S}{P}=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{49}+\frac{1}{50}}{50.\left(\frac{1}{50}+\frac{1}{49}+\frac{1}{48}+....+\frac{1}{2}\right)}=\frac{1}{50}$$

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Câu 20. **(HSG 7 thành phố Vinh, trường Nguyệt Ấn 2022 - 2023) Đề 378**

Cho $A=1+2^4+2^8+...+2^{1996}+2^{2000}$ và $B=1+2^2+2^4+...+2^{2000}+2^{2002}$. Tính tỉ số $\frac{A}{B}$

Lời giải

Ta có $A=1+2^4+2^8+...+2^{1996}+2^{2000}$

$$2^4.A=2^4+2^8+...+2^{2000}+2^{2004}$$

$$2^4.A-A=\left(2^4+2^8+...+2^{2000}+2^{2004}\right)-\left(1+2^4+2^8+...+2^{1996}+2^{2000}\right)$$

$$15A=2^{2004}-1$$

$$A=\frac{2^{2004}-1}{15}$$

Lại có: $B=1+2^2+2^4+...+2^{2000}+2^{2002}$.

$$2^2.B=2^2+2^4+...+2^{2002}+2^{2004}$$

$$2^2.B-B=\left(2^2+2^4+...+2^{2002}+2^{2004}\right)-\left(1+2^2+2^4+...+2^{2000}+2^{2002}\right)$$

$3B = 2^{2004} - 1$

$B = \frac{2^{2004} - 1}{3}$

$$\frac{A}{B} = \frac{\frac{2^{2004} - 1}{15}}{\frac{2^{2004} - 1}{3}} = \frac{3}{15} = \frac{1}{5}$$

Suy ra:

Vậy: $\frac{A}{B} = \frac{1}{5}$