SEQUENCE

1. The sequence a_n is defined as follows: $a_1 = 1$, $a_{n+1} = a_n + 1/a_n$ for $n \geq 1$. Prove that $a_{100} > 14$. (ASU 1968)
2. The sequence $a_1, a_2, \dots, a_n$ satisfies the following conditions: $a_1 = 0$, $|a_i| = |a_{i-1} + 1|$ for $i = 2, 3, \dots, n$. Prove that $(a_1 + a_2 + \dots + a_n)/n \geq -1/2$. (ASU 1968)
3. A sequence of finite sets of positive integers is defined as follows. $S_0 = \{m\}$, where $m > 1$. Then given S_n you derive S_{n+1} by taking k^2 and $k+1$ for each element k of S_n. For example, if $S_0 = \{5\}$, then $S_2 = \{7, 26, 36, 625\}$. Show that S_n always has 2^n distinct elements.(ASU 1972)
4. a_1 and a_2 are positive integers less than 1000. Define $a_n = \min\{|a_i - a_j| : 0 < i < j < n\}$. Show that $a_{21}=0$. (ASU 1976)
5. a_n is an infinite sequence such that $(a_{n+1} - a_n)/2$ tends to zero. Show that a_n tends to zero.(ASU1977)
6. Given a sequence $a_1, a_2, \dots, a_n$ of positive integers. Let S be the set of all sums of one or more members of the sequence. Show that S can be divided into n subsets such that the smallest member of each subset is at least half the largest member. (ASU 1977)
7. Show that there is an infinite sequence of reals $x_1, x_2, x_3, \dots$ such that $|x_n|$ is bounded and for any $m > n$, we have $|x_m - x_n| > 1/(m - n)$.(ASU 1978)
8. The real sequence $x_1 \geq x_2 \geq x_3 \geq \dots$ satisfies $x_1 + x_4/2 + x_9/3 + x_{16}/4 + \dots + x_N/n \leq 1$ for every square $N = n^2$. Show that it also satisfies $x_1 + x_2/2 + x_3/3 + \dots + x_n/n \leq 3$. (ASU1979)
9. Define the sequence a_n of positive integers as follows. $a_1 = m$. $a_{n+1} = a_n$ plus the product of the digits of a_n. For example, if $m = 5$, we have 5, 10, 10, ... Is there an m for which the sequence is unbounded?(ASU 1980)
10. The sequence a_n of positive integers is such that (1) $a_n \leq n^{3/2}$ for all n, and (2) m-n divides $k_m - k_n$ (for all $m > n$). Find a_n.(ASU 1981)
11. The sequence a_n is defined by $a_1 = 1$, $a_2 = 2$, $a_{n+2} = a_{n+1} + a_n$. The sequence b_n is defined by $b_1 = 2$, $b_2 = 1$, $b_{n+2} = b_{n+1} + b_n$. How many integers belong to both sequences?(ASU1982)
12. A subsequence of the sequence real sequence $a_1, a_2, \dots, a_n$ is chosen so that (1) for each i at least one and at most two of a_i, a_{i+1}, a_{i+2} are chosen and (2) the sum of the absolute values of the numbers in the subsequence is at least 1/6 $\sum_{i=1}^{n}|a_i|$.(ASU 1982)
13. a_n is the last digit of $[10^{n/2}]$. Is the sequence a_n periodic? b_n is the last digit of $[2^{n/2}]$. Is the sequence b_n periodic?(ASU 1983)
14. The real sequence x_n is defined by $x_1 = 1$, $x_2 = 1$, $x_{n+2} = x_{n+1}^2 - x_n/2$. Show that the sequence converges and find the limit.(ASU 1984)
15. The sequence $a_1, a_2, a_3, \dots$ satisfies $a_{4n+1} = 1$, $a_{4n+3} = 0$, $a_{2n} = a_n$. Show that it is not periodic.(ASU 1985)
16. The sequence of integers a_n is given by $a_0 = 0$, $a_n = p(a_{n-1})$, where p(x) is a polynomial whose coefficients are all positive integers. Show that for any two

positive integers m, k with greatest common divisor d, the greatest common divisor of a_m and a_k is a_d.(ASU 1988)

17. A sequence of positive integers is constructed as follows. If the last digit of a_n is greater than 5, then a_{n+1} is $9a_n$. If the last digit of a_n is 5 or less and a_n has more than one digit, then a_{n+1} is obtained from a_n by deleting the last digit. If a_n has only one digit, which is 5 or less, then the sequence terminates. Can we choose the first member of the sequence so that it does not terminate?(ASU 1991)
18. Define the sequence $a_1 = 1, a_2, a_3, \ldots$ by $a_{n+1} = a_1^2 + a_2^{\ 2} + a_3^2 + \ldots + a_n^2 + n$. Show that 1 is the only square in the sequence. (CIS 1992)
19. The sequence (a_n) satisfies $a_{m+n}+ a_{m-n}= \frac{1}{2}(a_{2m}+a_{2n})$ for all $m \geq n \geq 0$. If $a_1=1$, find a_{1995}. (Russian 1995)
20. The sequence $a_1, a_2, a_3, \ldots$ of positive integers is determined by its first two members and the rule $a_{n+2} = (a_{n+1} + a_n)/\gcd(a_n, a_{n+1})$. For which values of a_1 and a_2 is it bounded?(Russian 1999)
21. The sequence $a_1, a_2, \ldots, a_{3972}$ includes each of the numbers from 1 to 1986 twice. Can the terms be rearranged so that there are just n numbers between the two n's? (CMO 1986)
22. The integer sequence a_i is defined by $a_0 = m$, $a_1 = n$, $a_2 = 2n-m+2$, $a_{i+3} =3(a_{i+2} - a_{i+1}) + a_i$. It contains arbitrarily long sequences consecutive terms which are squares. Show that every term is a square.(CMO 1992)
23. $x_0, x_1, \ldots$, is a sequence of binary strings of length n. n is odd and $x_0 = 100\ldots01$. x_{m+1} is derived from x_m as follows: the kth digit in the string is 0 if the kth and k+1st digits in the previous string are the same, 1 otherwise. [The n+1th digit in a string means the 1st]. Show that if $x_m = x_n$, then m is a multiple of n].(CMO 1995)
24. $a_1, a_2, \ldots$ is a sequence of non-negative integers such that $a_{n+m} \leq a_n + a_m$ for all m, n. Show that if $N \geq n$, then $a_n + a_N \leq na_1 + N/n\ a_n$.(CMO 1997)
25. The sequence a_n is defined by $a_1 = 0$, $a_2 = 1$, $a_n = (n\ a_{n-1} + n(n-1)\ a_{n-2} + (-1)^{n-1}n)/2 + (-1)^n$. Find $a_n + 2\ nC1\ a_{n-1} + 3\ nC2\ a_{n-2} + \ldots + n\ nC(n-1)\ a_1$, where nCm is the binomial coefficient n!/(m! (n-m)!).(CMO 2000)
26. Let $a_1 = 0$, $a_{2n+1} = a_{2n} = n$. Let $s(n) = a_1 + a_2 + \ldots + a_n$. Find a formula for s(n) and show that $s(m + n) = mn + s(m - n)$ for $m > n$.(CanMO 1970)
27. Let $a_n = 1/(n(n+1))$. (1) Show that $1/n = 1/(n+1) + a_n$. (2) Show that for any integer $n > 1$ there are positive integers $r < s$ such that $1/n = a_r + a_{r+1} + \ldots + a_s$.(CanMO 1973)
28. Define the real sequence $a_1, a_2, a_3, \ldots$ by $a_1 = 1/2$, $n^2a_n = a_1 + a_2 + \ldots + a_n$. Evaluate a_n. (CanMO 1975)
29. The real sequence $x_0, x_1, x_2, \ldots$ is defined by $x_0 = 1$, $x_1 = 2$, $n(n+1)\ x_{n+1} = n(n-1)\ x_n - (n-2)\ x_{n-1}$. Find $x_0/x_1 + x_1x_2 + \ldots + x_{50}/x_{51}$.(CanMO 1976)
30. The real sequence $x_1, x_2, x_3, \ldots$ is defined by $x_1 = 1 + k$, $x_{n+1} = 1/x_n + k$, where $0 < k < 1$. Show that every term exceeds 1.(CanMO 1977)
31. Define the real sequence $x_1, x_2, x_3, \ldots$ by $x_1 = k$, where $1 < k < 2$, and $x_{n+1} = x_n - x_n^2/2 + 1$. Show that $|x_n - \sqrt{2}| < 1/2^n$ for $n > 2$.(CanMO 1985)

32. The integer sequence $a_1, a_2, a_3, \dots$ is defined by $a_1 = 39$, $a_2 = 45$, $a_{n+2} = a_{n+1}^2 - a_n$. Show that infinitely many terms of the sequence are divisible by 1986.(CanMO 1986)
33. Define two integer sequences $a_0, a_1, a_2, \dots$ and $b_0, b_1, b_2, \dots$ as follows. $a_0 = 0$, $a_1= 1$, $a_{n+2} = 4a_{n+1} - a_n$, $b_0 = 1$, $b_1 = 2$, $b_{n+2} = 4b_{n+1} - b_n$. Show that $b_n^2 = 3a_n^2 + 1$.(CanMO 1988)
34. A sequence of positive integers $a_1, a_2, a_3, \dots$ is defined as follows. $a_1 = 1$, $a_2 = 3$, $a_3 = 2$, $a_{4n} = 2a_{2n}$, $a_{4n+1} = 2a_{2n} + 1$, $a_{4n+2} = 2a_{2n+1} + 1$, $a_{4n+3} = 2a_{2n+1}$. Show that the sequence is a permutation of the positive integers. (CanMO 1993)
35. Show that non-negative integers $a \le b$ satisfy $(a^2 + b^2) = n^2(ab + 1)$, where n is a positive integer, if they are consecutive terms in the sequence a_k defined by $a_0 = 0$, $a_1 = n$, $a_{k+1} = n^2a_k - a_{k-1}$. (CanMO 1998)
36. Show that in any sequence of 2000 integers each with absolute value not exceeding 1000 such that the sequence has sum 1, we can find a subsequence of one or more terms with zero sum.(CanMO 2000)
37. Each member of the sequence $a_1, a_2, \dots, a_n$ belongs to the set $\{1, 2, \dots, n-1\}$ and $a_1 + a_2 + \dots + a_n < 2n$. Show that we can find a subsequence with sum n.(Irish 1988)
38. The sequence of nonzero reals $x_1, x_2, x_3, \dots$ satisfies $x_n = x_{n-2}x_{n-1}/(2x_{n-2} - x_{n-1})$ for all $n > 2$. For which (x_1, x_2) does the sequence contain infinitely many integral terms? (Irish 1988)
39. The sequence $a_1, a_2, a_3, \dots$ is defined by $a_1 = 1$, $a_{2n} = a_n$, $a_{2n+1} = a_{2n} + 1$. Find the largest value in $a_1, a_2, \dots, a_{1989}$ and the number of times it occurs.(Irish 1989)
40. The sequence $a_1, a_2, a_3, \dots$ is defined by $a_1 = 1$, $a_{2n} = a_n$, $a_{2n+1} = a_{2n} + 1$. Find the largest value in $a_1, a_2, \dots, a_{1989}$ and the number of times it occurs.(Irish 2002)
41. The sequence $\{x_n\}_{n=1}^{\infty}$ is defined as: $x_1=1$, $x_{n+1}=x_n^2- 3x_n+ 4$, n= 1,2,3,...

    a) Prove that $\{x_n\}_{n=1}^{\infty}$ is monotone increasing and unbounded.

    b) Prove that the sequence $\{y_n\}_{n=1}^{\infty}$ defined as $y_n = 1/(x_1-1) +\dots+1/(x_n-1)$ is convergent and find its limit (Bungari 1997-Problem in winter)
42. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of integer number such that their dicemal representations consist of even digits($a_1=2$, $a_2=4$, $a_3=6$,...). Find all integer number *m* such that $a_m= 12m$.(Bungari 1998 - Problem in winter)
43. Prove that for every positive number a the sequence $\{x_n\}_{n=1}^{\infty}$ such that $x_1=1$, $x_2=a$, $x_{n+2}=\sqrt[3]{x_{n+1}^2 x_n}$,$n\ge 1$ is convergent and find its limit.(Bungari 2000-Problem11.1)
44. Given the sequence $x_n= a\sqrt{n^2+1}$, n=1,2,.....where a is a real number:

    a) Find the values of a such that the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent.

    b) Find the values of a such that the sequence $\{x_n\}_{n=1}^{\infty}$ is monotone increasing. (Bungari 1999-Pro in winter)
45. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence such that $x_1=43$, $x_2=142$, $x_{n+2}= 3x_{n+1} + x_n$,$n\ge 1$.Prove that:

    a) x_n and x_{n+1} are relatively prime for all n.

    b) for every natural number *m* there exits infinitely many natural number n such that x_n-1 and x_{n+1}-1 both divisible by *m*. (Bungari 2000-Pro3 third round)

46. A sequence is $a_1, a_2, a_3,....$ is defined by $a_1= k$, $a_2= 5k-2$ and $a_{n+2}= 3a_{n+1}- 2a_n$, $n \geq 1$, where k is a real number
a)Find all values of k, such that the sequence $\{a_n\}_{n=1}^{\infty}$ is convergent.
b)Prove that if k=1 then: $a_{n+2} = \left[\dfrac{7a_{n+1}^2 - 8a_n a_{n+1}}{1 + a_n + a_{n+1}} \right]$,$n \geq 1$, where $\lfloor x \rfloor$ denoted the integer part of x.(Bungari 2001,2-4)
47. Define the sequence $a_1, a_2, a_3, ...$ by $a_1 = 1$, $a_n = a_{n-1} - n$ if $a_{n-1} > n$, $a_{n-1} + n$ if $a_{n-1} \leq n$. Let S be the set of n such that $a_n = 1993$. Show that S is infinite. Find the smallest member of S. If the element of S are written in ascending order show that the ratio of consecutive terms tends to 3.(IMO SHORTLIST 1993)
48. The sequence $x_0, x_1, x_2, ...$ is defined by $x_0 = 1994$, $x_{n+1} = x_n^2/(x_n + 1)$. Show that $[x_n] = 1994 - n$ for $0 \leq n \leq 998$.(IMO SHORTLIST 1994)
49. Define the sequences a_n, b_n, c_n as follows. $a_0 = k$, $b_0 = 4$, $c_0 = 1$. If a_n is even then $a_{n+1} = a_n/2$, $b_{n+1} = 2b_n$, $c_{n+1} = c_n$. If a_n is odd, then $a_{n+1} = a_n - b_n/2 - c_n$, $b_{n+1} = b_n$, $c_{n+1} = b_n + c_n$. Find the number of positive integers $k < 1995$ such that some $a_n = 0$. (IMO SHORTLIST 1994)
50. Define the sequence $a_1, a_2, a_3, ...$ as follows. a_1 and a_2 are coprime positive integers and $a_{n+2} = a_{n+1}a_n + 1$. Show that for every $m > 1$ there is an $n > m$ such that a_m^m divides a_n^n. Is it true that a_1 must divide a_n^n for some $n > 1$?(IMO SHORTLIST 1994)
51. Find a sequence f(1), f(2), f(3), ... of non-negative integers such that 0 occurs in the sequence, all positive integers occur in the sequence infinitely often, and $f(f(n^{163})) = f(f(n)) + f(f(361))$.(IMO SHORTLIST 1995)
52. Given $a > 2$,define the sequence $a_0,a_1,a_2, ...$by $a_0 = 1$, $a_1 = a$, $a_{n+2} = a_{n+1}(a_{n+1}^2/a_n^2 -2)$. Show that $1/a_0 + 1/a_1 + 1/a_2 + ... + 1/a_n < 2 + a - (a^2 - 4)^{1/2}$.(IMO SHORTLIST 1996)
53. The sequence $a_1, a_2, a_3, ...$ is defined by $a_1 = 0$ and $a_{4n} = a_{2n} + 1$, $a_{4n+1} = a_{2n} - 1$, $a_{4n+2} = a_{2n+1} - 1$, $a_{4n+3} = a_{2n+1} + 1$. Find the maximum and minimum values of a_n for n = 1, 2, ... , 1996 and the values of n at which they are attained. How many terms a_n for n = 1, 2, ... , 1996 are 0? (IMO SHORTLIST 1996)
54. A finite sequence of integers $a_0, a_1, ... , a_n$ is called *quadratic* if $|a_1 - a_0| = 1^2$, $|a_2 - a_1| = 2^2$,..., $|a_n - a_{n-1}| = n^2$. Show that any two integers h, k can be linked by a quadratic sequence (in other words for some n we can find a quadratic sequence a_i with $a_0 = h$, $a_n = k$). Find the shortest quadratic sequence linking 0 and 1996. (IMO SHORTLIST 1996)
55. The sequences R_n are defined as follows. $R_1 = (1)$. If $R_n = (a_1, a_2, ... , a_m)$, then $R_{n+1} = (1, 2, ... , a_1, 1, 2, ... , a_2, 1, 2, ... , 1, 2, ... , a_m, n+1)$. For example, $R_2 = (1, 2)$, $R_3 = (1, 1, 2, 3)$, $R_4 = (1, 1, 1, 2, 1, 2, 3, 4)$. Show that for $n > 1$, the kth term from the left in R_n is 1 iff the kth term from the right is not 1.(IMO SHORTLIST 1997)
56. The sequence $a_1, a_2, a_3, ...$ is defined as follows. $a_1 = 1$. a_n is the smallest integer greater than a_{n-1} such that we cannot find $1 \leq i, j, k \leq n$ (not necessarily distinct) such that $a_i + a_j = 3a_k$. Find a_{1998}. (IMO SHORTLIST 1998)

57. The sequence $0 \le a_0 < a_1 < a_2 < \dots$ is such that every non-negative integer can be uniquely expressed as $a_i + 2a_j + 4a_k$ (where i, j, k are not necessarily distinct). Find a_{1998}. (IMO SHORTLIST 1998)
58. Let $p > 3$ be a prime. Let h be the number of sequences $a_1, a_2, \dots, a_{p-1}$ such that $a_1 + 2a_2 + 3a_3 + \dots + (p-1)a_{p-1}$ is divisible by p and each a_i is 0, 1 or 2. Let k be defined similarly except that each a_i is 0, 1 or 3. Show that $h \le k$ with equality if $p = 5$. (IMO SHORTLIST 1999)
59. Show that there exist two strictly increasing sequences $a_1, a_2, a_3, \dots$ and $b_1, b_2, b_3, \dots$ such that $a_n(a_n + 1)$ divides $b_n^2 + 1$ for each n.(IMO SHORTLIST 1999)
60. $0 = a_0 < a_1 < a_2 < \dots$ and $0 = b_0 < b_1 < b_2 < \dots$ are sequences of real numbers such that: (1) if $a_i + a_j + a_k = a_r + a_s + a_t$, then (i, j, k) is a permutation of (r, s, t); and (2) a positive real x can be represented as $x = a_j - a_i$ iff it can be represented as $b_m - b_n$. Prove that $a_k = b_k$ for all k. (IMO SHORTLIST 2000)
61. Find all finite sequences $a_0, a_1, a_2, \dots, a_n$ such that a_m equals the number of times that m appears in the sequence.(IMO SHORTLIST 2001)
62. The sequence a_n is defined by $a_1= 11^{11}$, $a_2 = 12^{12}$, $a_3 = 13^{13}$, and $a_{n+3} = |a_{n+2} - a_{n+1}| + |a_{n+1} - a_n|$. Find a_n, where $n = 14^{14}$.(IMO SHORTLIST 2001)
63. The infinite real sequence $x_1, x_2, x_3, \dots$ satisfies $|x_i - x_j| \ge 1/(i + j)$ for all unequal i, j. Show that if all x_i lie in the interval [0, c], then $c \ge 1$.(IMO SHORTLIST 2002)
64. The sequence a_n is defined by $a_1 = a_2 = 1$, $a_{n+2} = a_{n+1} + 2a_n$. The sequence b_n is defined by $b_1 = 1$, $b_2 = 7$, $b_{n+2} = 2b_{n+1} + 3b_n$. Show that the only integer in both sequences is 1. (USAMO 1973)
65. $a_1, a_2, \dots, a_n$ is an arbitrary sequence of positive integers. A member of the sequence is picked at random. Its value is a. Another member is picked at random, independently of the first. Its value is b. Then a third, value c. Show that the probability that $a + b + c$ is divisible by 3 is at least 1/4.(USAMO 1979)
66. $0 < a_1 \le a_2 \le a_3 \le \dots$ is an unbounded sequence of integers. Let $b_n = m$ if a_m is the first member of the sequence to equal or exceed n. Given that $a_{19} = 85$, what is the maximum possible value of $a_1 + a_2 + \dots + a_{19} + b_1 + b_2 + \dots + b_{85}$?(USAMO 1985)
67. $a_1, a_2, \dots, a_n$ is a sequence of 0s and 1s. T is the number of triples (a_i, a_j, a_k) with $i < j < k$ which are not equal to (0, 1, 0) or (1, 0, 1). For $1 \le i \le n$, f(i) is the number of $j < i$ with $a_j = a_i$ plus the number of $j > i$ with $a_j \ne a_i$. Show that $T = f(1) (f(1) - 1)/2 + f(2) (f(2) - 1)/2 + \dots + f(n) (f(n) - 1)/2$. If n is odd, what is the smallest value of T?(USAMO 1987)
68. The sequence a_n of odd positive integers is defined as follows: $a_1 = r$, $a_2 = s$, and a_n is the greatest odd divisor of $a_{n-1} + a_{n-2}$. Show that, for sufficiently large n, a_n is constant and find this constant (in terms of r and s).(USAMO 1993)
69. The sequence $a_1, a_2, \dots, a_{99}$ has $a_1 = a_3 = a_5 = \dots = a_{97} = 1$, $a_2 = a_4 = a_6 = \dots = a_{98} = 2$, and $a_{99} = 3$. We interpret subscripts greater than 99 by subtracting 99, so that a_{100} means a_1 etc. An allowed move is to change the value of any one of the a_n to another member of {1, 2, 3} different from its two neighbors, a_{n-1} and a_{n+1}. Is there a sequence of allowed moves which results in $a_m = a_{m+2} = \dots = a_{m+96} = 1$, $a_{m+1} = a_{m+3} = \dots = a_{m+95} = 2$, $a_{m+97} = 3$, $a_{n+98} = 2$ for some m? [So if m = 1, we have just interchanged the values of a_{98} and a_{99}.](USAMO 1994)

70. x_i is a infinite sequence of positive reals such that for all n, $x_1 + x_2 + ... + x_n \geq \sqrt{n}$. Show that $x_1^2 + x_2^2 + ... + x_n^2 > (1 + 1/2 + 1/3 + ... + 1/n) / 4$ for all n.(USAMO 1994)
71. $a_0, a_1, a_2, ...$ is an infinite sequence of integers such that $a_n - a_m$ is divisible by n - m for all (unequal) n and m. For some polynomial p(x) we have $p(n) > |a_n|$ for all n. Show that there is a polynomial q(x) such that $q(n) = a_n$ for all n.(USAMO 1995)
72. A type 1 sequence is a sequence with each term 0 or 1 which does not have 0, 1, 0 as consecutive terms. A type 2 sequence is a sequence with each term 0 or 1 which does not have 0, 0, 1, 1 or 1, 1, 0, 0 as consecutive terms. Show that there are twice as many type 2 sequences of length n+1 as type 1 sequences of length n.(USAMO 1996)
73. Let p_n be the nth prime. Let 0 < a < 1 be a real. Define the sequence x_n by $x_0 = a$, x_n = the fractional part of p_n/x_{n-1} if x_n ¹ 0, or 0 if $x_{n-1} = 0$. Find all a for which the sequence is eventually zero.(USAMO 1997)
74. A sequence of polygons is derived as follows. The first polygon is a regular hexagon of area 1. Thereafter each polygon is derived from its predecessor by joining two adjacent edge midpoints and cutting off the corner. Show that all the polygons have area greater than 1/3.(USAMO 1997)
75. The sequence of non-negative integers $c_1, c_2, ... , c_{1997}$ satisfies $c_1 \geq 0$ and $c_m + c_n \geq c_{m+n} \geq c_m + c_n + 1$ for all m, n > 0 with m + n < 1998. Show that there is a real k such that $c_n = [nk]$ for $1 \geq n \geq 1997$. (USAMO 1997)
76. Define the sequence a_n, by $a_1 = 0$, $a_2 = 1$,$a_3= 2$,$a_4 = 3$, and $a_{2n} = a_{2n-5} + 2^n$, $a_{2n+1} = a_{2n} + 2^{n-1}$. Show that $a_{2n} = [17/7\ 2^{n-1}] - 1$, $a_{2n-1} = [12/7\ 2^{n-1}] - 1$.(BMO 1972)
77. Define sequences of integers by $p_1 = 2$, $q_1 = 1$, $r_1 = 5$, $s_1= 3$, $p_{n+1} = p_n^2 + 3\ q_n^2$, $q_{n+1} = 2\ p_nq_n$, $r_n = p_n + 3\ q_n$, $s_n = p_n + q_n$. Show that $p_n/q_n > \sqrt{3} > r_n/s_n$ and that p_n/q_n differs from $\sqrt{3}$ by less than $s_n/(2\ r_nq_n^2)$.(BMO 1972)
78. Show that there is a unique sequence $a_1, a_2, a_3, ...$ such that $a_1 = 1$, $a_2 > 1$, $a_{n+1}a_{n-1} = a_n^3 + 1$, and all terms are integral.(BMO 1978)
79. Find all real a_0 such that the sequence $a_0, a_1, a_2, ...$ defined by $a_{n+1} = 2^n - 3a_n$ has $a_{n+1} > a_n$ for all $n \geq 0$.(BMO 1980)
80. The sequence $u_0, u_1, u_2, ...$ is defined by $u_0 = 2$, $u_1 = 5$, $u_{n+1}u_{n-1} - u_n^2 = 6^{n-1}$. Show that all terms of the sequence are integral. (BMO 1981)
81. The sequence $p_1, p_2, p_3, ...$ is defined as follows. $p_1 = 2$. p_{n+1} is the *largest* prime divisor of $p_1p_2 ... p_n + 1$. Show that 5 does not occur in the sequence.(BMO 1982)
82. Let { x } denote the nearest integer to x, so that x - 1/2 $\geq$ { x } < x + 1/2. Define the sequence $u_1, u_2, u_3, ...$ by $u_1 = 1$. $u_{n+1} = u_n + \{ u_n\sqrt{2} \}$. So, for example, $u_2 = 2$, $u_3 = 5$, $u_3 = 12$. Find the units digit of u_{1985}.(BMO 1985)
83. The real sequence $x_1, x_1, x_2, ...$ is defined by $x_0 = 1$, $x_{n+1} = (3x_n + \sqrt{5x_n^2 - 4})/2$. Show that all the terms are integers.(BMO 2002)
84. A sequence of values in the range 0, 1, 2, ... , k-1 is defined as follows: $a_1 = 1$, $a_n = a_{n-1} + n \pmod k$. For which k does the sequence assume all k possible values? (APMO 1991)
85. $a_1, a_2, a_3, ... a_n$ is a sequence of non-zero integers such that the sum of any 7 consecutive terms is positive and the sum of any 11 consecutive terms is negative. What is the largest possible value for n?(APMO 1992)

86. Find all real sequences $x_1, x_2, \ldots, x_{1995}$ which satisfy $2\sqrt{x_n - n + 1} \geq x_{n+1} - n + 1$ for n = 1, 2, ... , 1994, and $2\sqrt{x_{1995} - 1994} \geq x_1 + 1$.(APMO 1995)
87. Find the smallest n such that any sequence $a_1, a_2, \ldots, a_n$ whose values are relatively prime square-free integers between 2 and 1995 must contain a prime. [An integer is square-free if it is not divisible by any square except 1.](APMO 1995)
88. P_1 and P_3 are fixed points. P_2 lies on the line perpendicular to P_1P_3 through P_3. The sequence $P_4, P_5, P_6, \ldots$ is defined inductively as follows: P_{n+1} is the foot of the perpendicular from P_n to $P_{n-1}P_{n-2}$. Show that the sequence converges to a point P (whose position depends on P_2). What is the locus of P as P_2 varies?(APMO 1997)
89. The integers r, s are non-zero and k is a positive real. The sequence a_n is defined by $a_1 = r$, $a_2 = s$, $a_{n+2} = (a_{n+1}^2 + k)/a_n$. Show that all terms of the sequence are integers iff $(r^2 + s^2 + k)/(rs)$ is an integer.(Balkan 1986)
90. x_n is the sequence 51, 53, 57, 65, ... , $2^n + 49$, ... Find all n such that x_n and x_{n+1} are each the product of just two distinct primes with the same difference.(Balkan 1988)
91. The sequence u_n is defined by $u_1 = 1$, $u_2 = 3$, $u_n = (n+1)\, u_{n-1} - n\, u_{n-2}$. Which members of the sequence which are divisible by 11? (Balkan 1990)
92. Define a_n by $a_3 = (2 + 3)/(1 + 6)$, $a_n = (a_{n-1} + n)/(1 + n\, a_{n-1})$. Find a_{1995}. (Balkan 1995)
93. $0 = a_1, a_2, a_3, \ldots$ is a non-decreasing, unbounded sequence of non-negative integers. Let the number of members of the sequence not exceeding n be b_n. Prove that $(x_0 + x_1 + \ldots + x_m)(y_0 + y_1 + \ldots + y_n) \geq (m + 1)(n + 1)$.(Balkan 1999)
94. The sequence a_n is defined by $a_1 = 20$, $a_2 = 30$, $a_{n+1} = 3a_n - a_{n-1}$. Find all n for which $5a_{n+1}a_n + 1$ is a square.(Balkan 2002)
95. a_i and b_i are real, and $S_1^\infty a_i^2$ and $S_1^\infty b_i^2$ converge. Prove that $S_1^\infty (a_i - b_i)^p$ converges for $p \geq 2$.(Putnam 1940)
96. The sequence a_n of real numbers satisfies $a_{n+1} = 1/(2 - a_n)$. Show that $\lim_{n\to\infty} a_n = 1$. (Putnam 1947)
97. a_n is a sequence of positive reals decreasing monotonically to zero. b_n is defined by $b_n = a_n - 2a_{n+1} + a_{n+2}$ and all b_n are non-negative. Prove that $b_1 + 2b_2 + 3b_3 + \ldots = a_1$. (Putnam 1948)
98. a_n is a sequence of positive reals. Show that $\limsup_{n\to\infty}((a_1 + a_{n+1})/a_n)^n \geq e$.(Putam 1949)
99. The sequences a_n, b_n, c_n of positive reals satisfy: (1) $a_1 + b_1 + c_1 = 1$; (2) $a_{n+1} = a_n^2 + 2b_nc_n$, $b_{n+1} = b_n^2 + 2c_na_n$, $c_{n+1} = c_n^2 + 2a_nb_n$. Show that each of the sequences converges and find their limits. (Putnam 1947)
100. The sequence a_n is defined by $a_0 = \alpha$, $a_1 = \beta$, $a_{n+1} = a_n + (a_{n-1} - a_n)/(2n)$. Find $\lim_{n\to\infty} a_n$. (Putnam 1950)
101. Let $a_n = S_1^n (-1)^{i+1}/i$. Assume that $\lim_{n\to\infty} a_n = k$. Rearrange the terms by taking two positive terms, then one negative term, then another two positive terms, then another negative term and so on. Let b_n be the sum of the first n terms of the

rearranged series. Assume that $\lim_{n\to\infty} b_n = h$. Show that $b_{3n} = a_{4n} + a_{2n}/2$, and hence that $h \neq k$.(Putnam 1954)

102. Let a be a positive real. Let $a_n = S_1^n (a/n + i/n)^n$. Show that $\lim_{n\to\infty} a_n \in (e^a, e^{a+1})$. (Putnam 1954)

103. a_n is a sequence of monotonically decreasing positive terms such that Σa_n converges. S is the set of all Σb_n, where b_n is a subsequence of a_n. Show that S is an interval iff $a_{n-1} \leq \Sigma_n^\infty a_i$ for all n.(Putnam 1955)

104. The sequence a_n is defined by $a_1 = 2$, $a_{n+1} = a_n^2 - a_n + 1$. Show that any pair of values in the sequence are relatively prime and that $\sum \frac{1}{a_n} = 1$.(Putnam 1956)

105. Define a_n by $a_1 = \ln a$, $a_2 = \ln(a - a_1)$, $a_{n+1} = a_n + \ln(a - a_n)$. Show that $\lim_{n\to\infty} a_n = a-1$. (Putnam 1957)

106. The sequence a_n is defined by its initial value a_1, and $a_{n+1} = a_n(2 - k\, a_n)$. For what real a_1 does the sequence converge to 1/k?(Putnam 1957)

107. A sequence of numbers $a_i \in [0, 1]$ is chosen at random. Show that the expected value of n, where $S_1^n a_i > 1$, $S_1^{n-1} a_i \leq 1$ is e.(Putnam 1958)

108. a and b are positive irrational numbers satisfying $1/a + 1/b = 1$. Let $a_n = [n\, a]$ and $b_n = [n\, b]$, for n = 1, 2, 3, ... Show that the sequences a_n and b_n are disjoint and that every positive integer belongs to one or the other.(Putnam 1959)

109. The sequence $a_1, a_2, a_3, \ldots$ of positive integers is strictly monotonic increasing, $a_2 = 2$ and $a_{mn} = a_m a_n$ for m, n relatively prime. Show that $a_n = n$. (Putnam 1963)

110. Show that for any sequence of positive reals, a_n, we have $\lim \sup_{n\to\infty} \left(n \frac{a_{n+1} + 1}{a_n} - 1 \right) \geq 1$. Show that we can find a sequence where equality holds. (Putnam 1963)

111. The series $\sum_{n=1}^{\infty} a_n$ of non-negative terms converges and $a_i <= 100 a_n$ for i = n, n + 1, n + 2, ... , 2n. Show that $\lim_{n\to\infty} n a_n = 0$.(Putnam 1963)

112. The sequence of integers u_n is bounded and satisfies $u_n = (u_{n-1} + u_{n-2} + u_{n-3}u_{n-4})/(u_{n-1}u_{n-2} + u_{n-3} + u_{n-4})$. Show that it is periodic for sufficiently large n.(Putnam 1964)

113. a_n are positive integers such that $\Sigma\, 1/a_n$ converges. b_n is the number of a_n which are <= n. Prove $\lim b_n/n = 0$.(Putnam 1964)

114. Let a_n be a strictly monotonic increasing sequence of positive integers. Let b_n be the least common multiple of $a_1, a_2, \ldots, a_n$. Prove that $\Sigma\, 1/b_n$ converges. (putnam 1964)

115. $\{a_n\}_{n=1}^{\infty}$ is an infinite sequence of real numbers. Let $b_n = 1/n \sum_{i=1}^{n} \exp(ia_i)$. Prove that $b_1, b_2, b_3, b_4, \ldots$ converges to k if $b_1, b_4, b_9, b_{16}, \ldots$ converges to k. (Putnam1965)

116. Define the sequence $\{a_n\}_{n=1}^{\infty}$ by $a_1 \in (0, 1)$, and $a_{n+1} = a_n(1 - a_n)$. Show that $\lim_{n\to\infty} na_n = 1$. (Putnam 1966)

117. a_n is a sequence of positive reals such that $\sum_{n=1}^{\infty} 1/a_n$ converges. Let $s_n = \sum_{i=1}^{n} a_i$. Prove that $\sum_{n=1}^{\infty} n^2a_n/s_n^2$ converges.(Putnam 1966)

118. Let u_n be the number of symmetric n x n matrices whose elements are all 0 or 1, with exactly one 1 in each row. Take $u_0 = 1$. Prove $u_{n+1} = u_n + n\, u_{n-1}$ and $\sum_{n=0}^{\infty} u_n x^n/n! = e^{f(x)}$, where $f(x) = x + (1/2)\, x^2$.(Putnam 1967)

119. We are given a sequence $a_1, a_2, \ldots, a_n$. Each a_i can take the values 0 or 1. Initially, all $a_i = 0$. We now successively carry out steps 1, 2, ... , n. At step m we change the value of a_i for those i which are a multiple of m. Show that after step n, $a_i = 1$ if i is a square. Devise a similar scheme to give $a_i = 1$ if i is twice a square. (Putnam 1967)

120. The sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1a_2 = 1$, $a_2a_3 = 2$, $a_3a_4 = 3$, $a_4a_5 = 4$, ... Also, $\lim_{n\to\infty} a_n/a_{n+1} = 1$. Prove that $a_1 = \sqrt{\frac{2}{\pi}}$.(Putnam 1969)

121. The sequence a_i, i = 1, 2, 3, ... is strictly monotonic increasing and the sum of its inverses converges. Let f(x) = the largest i such that $a_i < x$. Prove that f(x)/x tends to 0 as x tends to infinity.(Putnam 1969)

122. The real sequence $a_1, a_2, a_3, \ldots$ has the property that $\lim_{n\to\infty} (a_{n+2} - a_n) = 0$. Prove that $\lim_{n\to\infty} (a_{n+1} - a_n)/n = 0$.(Putnam 1970)

123. A sequence $\{x_n\}_{n=1}^{\infty}$ is said to have a Cesaro limit if $\lim_{n\to\infty} x_1 + x_2 + \ldots + x_n)/n$ exists. Find all (real-valued) functions f on the closed interval [0, 1] such that $\{ f(x_i) \}$ has a Cesaro limit if $\{x_n\}_{n=1}^{\infty}$ has a Cesaro limit.(Putnam 1972)

124. $a_n = \pm 1/n$ and $a_{n+8} > 0$ if $a_n > 0$. Show that if four of $a_1, a_2, \ldots, a_8$ are positive, then $\sum_{n=1}^{\infty} a_n$ converges. Is the converse true?(Putnam 1973)

125. Let $0 < \alpha < 1/4$. Define the sequence p_n by $p_0 = 1$, $p_1 = 1 - \alpha$, $p_{n+1} = p_n - \alpha\, p_{n-1}$. Show that if each of the events $A_1, A_2, \ldots, A_n$ has probability at least $1 - \alpha$, and A_i and A_j are independent for $| i - j | > 1$, then the probability of all A_i occurring is at least p_n. You may assume that all p_n are positive.(Putnam 1976)

126. a_n are defined by $a_1 = \alpha$, $a_2 = \beta$, $a_{n+2} = a_na_{n+1}/(2a_n - a_{n+1})$. α, β are chosen so that $a_{n+1} \neq 2a_n$. For what α, β are infinitely many a_n integral?(Putnam 1979)

127. Define a_n by $a_0 = \alpha$, $a_{n+1} = 2a_n - n^2$. For which α are all a_n positive? (Putnam 1980)

128. Let $f(n) = n + [\sqrt{n}]$. Define the sequence a_i by $a_0 = m$, $a_{n+1} = f(a_n)$. Prove that it contains at least one square.(Putnam 1983)

129. Define a sequence of convex polygons P_n as follows. P_0 is an equilateral triangle side 1. P_{n+1} is obtained from P_n by cutting off the corners one-third of the

way along each side (for example P_1 is a regular hexagon side 1/3). Find $\lim_{n=1\to\infty}$ area(P_n). (Putnam 1984)

130. Let a_n be the sequence defined by $a_1 = 3$, $a_{n+1} = 3^k$, where $k = a_n$. Let b_n be the remainder when a_n is divided by 100. Which values b_n occur for infinitely many n? (Putnam 1985)

131. Prove that the sequence $a_0 = 2, 3, 6, 14, 40, 152, 784, \ldots$ with general term $a_n = (n+4)\, a_{n-1} - 4n\, a_{n-2} + (4n-8)\, a_{n-3}$ is the sum of two well-known sequences. (Putnam 1990)

132. Let S be the set of points (x, y) in the plane such that the sequence a_n defined by $a_0 = x$, $a_{n+1} = (a_n^2 + y^2)/2$ converges. What is the area of S?(Putnam 1992)

133. The sequence a_n of non-zero reals satisfies $a_n^2 - a_{n-1}a_{n+1} = 1$ for $n \ge 1$. Prove that there exists a real number α such that $a_{n+1} = \alpha\, a_n - a_{n-1}$ for $n \ge 1$.(Putnam 1993)

134. Let $a_0, a_1, a_2, \ldots$ be a sequence such that: $a_0 = 2$; each $a_n = 2$ or 3; a_n = the number of 3s between the nth and n+1th 2 in the sequence. So the sequence starts: 233233323332332 ... . Show that we can find α such that $a_n = 2$ if $n = [\alpha m]$ for some integer $m \ge 0$. (Putnam 1993)

135. a_n is a sequence of positive reals satisfying $a_n <= a_{2n} + a_{2n+1}$ for all n. Prove that $\Sigma\ a_n$ diverges.(Putnam 1994)

136. Define the sequence a_n by $a_1 = 2$, $a_{n+1} = 2^{a_n}$. Prove that $a_n \equiv a_{n-1}$ (mod n) for $n \ge 2$. (Putnam 1997)

137. Define the sequence of decimal integers a_n as follows: $a_1 = 0$; $a_2 = 1$; a_{n+2} is obtained by writing the digits of a_{n+1} immediately followed by those of a_n. When is a_n a multiple of 11?(Putnam 1998)

138. k is a positive constant. The sequence x_i of positive reals has sum k. What are the possible values for the sum of x_i^2 ?(Putnam 2000)

139. $x_1 < x_2 < x_3 < \ldots$ is a sequence of positive reals such that $\lim x_n/n = 0$. Is it true that we can find arbitrarily large N such that all of $(x_1 + x_{2N-1})$, $(x_2 + x_{2N-2})$, $(x_3 + x_{2N-3})$, ... , $(x_{N-1} + x_{N+1})$ are less than $2\, x_N$?(Putnam 2001)

140. The sequence u_n is defined by $u_0 = 1$, $u_{2n} = u_n + u_{n-1}$, $u_{2n+1} = u_n$. Show that for any positive rational k we can find n such that $u_n/u_{n+1} = k$.(Putnam 2002)

141. The sequence $\{a_n\}_{n=1}^{\infty}$ is defined by $a_1=1$, $a_{n+1}=\dfrac{a_n}{n}+\dfrac{n}{a_n}$, $n\ge1$. Prove that $\lfloor a_n^2 \rfloor = n$ when $n \ge 4$ (it is denoted by $\lfloor x \rfloor$ the integer part of the number x). (Bungari 1996- round 4)

142. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of integer number such that $(n-1)a_{n+1} = (n+1)a_n - 2(n-1)$ for any $n\ge1$. If 2000 divides a_{1999}, find the smallest $n \ge 2$ such that 2000 divides a_n.(Bungari 1994 -round 4)

143. An integer sequence satisfies $a_{n+1}=a_n^3+1999$. Show that it contains at most one square.(APMC 1999)

144. Define a sequence $a_{n}\ge1$ by $a_1=1, a_2=2$ and $a_{n+2}=2a_{n+1} - a_{n+2}$ for $n\ge 1$. Prove that for any m, $a_m a_{m+1}$ is also a term in the sequence.(INDIAN 1996)

145. Let $a_1=2$, $a_2=5$ and $a_{n+2}=(2-n^2)a_{n+1}+(2+n^2)a_n$ for $n\ge1$. Do there exist p,q,r so that $a_p a_q = a_r$.(Czech-Slovak1995)

146. Defined a sequence by $x_0, x_1, \in R$ and $x_{n+2} = \frac{1+x_{n+1}}{x_n}$ for $n \geq 0$. Find x_{1998}. (Ireland 1998)

147. Defined sequences $x_1, x_2, \ldots\ldots, y_1, y_2, \ldots..$ by $x_1 = y_1 = \sqrt{3}$ and $x_{n+1} = x_n + \sqrt{1+x_n^2}$, $y_{n+1} = \frac{y_n}{1+\sqrt{1+y_n^2}}$. Prove that for $n \geq 2$ we have $2 < x_n y_n < 3$.(Belarus 1999)

148. Consider a finite sequence $(a_n) \subset N$ so that any two distinct sub sequences have different sums. Prove that $\sum_{k=1}^{n} \frac{1}{a_k} < 2$.(Romania 1999)

149. Let $x_1 > 0$ and $x_{n+1} \geq (n+2)x_n - \sum_{n=1}^{n-1} kx_k$ for $n \geq 2$. Prove that for any $a \in R$ the sequence (x_n) even tually gets bigger than *a*. (Romania 1999)

150. Let $n \geq 3$ be an integer, and suppose that the sequence $a_1, a_2, \ldots., a_n$ satisfies $a_{i-1} + a_{i+1} = k_i a_i$ for positive integer k_i. Prove that $2n \leq \sum_{i=1}^{n} k_i \leq 3n$. (Taiwan1997)

151. Find all sequence $a_1, a_2, \ldots, a_{2000}$ of real number such that $\sum_{i=1}^{2000} a_i = 1999$ and for any $n \geq 1$ we have $1/2 < a_n < 1$ and $a_{n+1} = a_n(2 - a_n)$. (Turkey 2000)

152. Prove that for any positive integer a_1 there is an increasing sequence of integers $a_1, a_2, \ldots.$so that for any natural number k we have $a_1 + \ldots + a_k$ divide $a_1^2 + \ldots + a_k^2$. (Russian 1995)

153. Let (x_n) be the sequence of natural number such that: $x_1 = 1$ and $x_n < x_{n+1} \leq 2n$ for $1 \leq n$. Prove that for every natural number k, there exist the subscripts r and s, such that $x_r - x_s = k$.(Poland 1993)

154. The sequence (x_n) is given by $x_1 = 1/2$, $x_n = \frac{2n-3}{2n} x_{n-1}$ for n=2,3,.....Prove that for all natural number $1 \leq n$ the following inequality holds $x_1 + x_2 + \ldots.. + x_n < 1$. (Poland 1995)

155. Given a sequence $a_1, a_2, \ldots, a_{99}$ of one-digit numbers with the poperty that if for some n we have $a_1 = 1$, then $a_{n+1} \neq 2$; and if for some n we have $a_n = 3$, then $a_{n+1} \neq 4$. Prove that exist two number $k, l \in \{1, 2, \ldots, 98\}$ such that $a_k = a_l$ and $a_{k+1} = a_{l+1}$.(Poland 1996-2nd)

156. Given an integer $n \geq 2$ and positive number $x_1, x_2, \ldots., x_n$ with the sum equal to 1. a) Prove that for any positive number $a_1, \ldots, a_n$ with the sum equal to 1, hold the following inequality: $2\sum_{i<j} a_i a_j \leq \frac{n-2}{n-1} + \sum_{i=1}^{n} \frac{x_i a_i^2}{1-x_i}$. b) Determine all number $a_1, \ldots, a_n$ for which the above inequality turns into the equality. (Poland 1996-3rd)

157. For a natural number $k\geq 1$ denote by $p(k)$ the least prime number which is not a divisor of k. If $p(k)>2$, then we define $q(k)$ to be the product of all primes less than $p(k)$; if $p(k)=2$, we put $q(k)=1$. define the sequence (x_n) by the formulas $x_0=1$, $x_{n+1}=\frac{x_n p(x_n)}{q(x_n)}$ for $n\geq 0$. Determine all positive integers n with $x_n=111111$. (Poland 1996-3rd)

158. Positve integers $x_1,...,x_7$ satisfy the conditions:$x_6=144$ and $x_{n+3}=x_{n+2}(x_{n+1}+x_n)$ for $n\geq 1$. Determine x_7.(Poland 1997-3rd)

159. The sequence $a_1,a_2,...$is defined by $a_1=0$, $a_n=a_{\lfloor n/2\rfloor}+(-1)^{n(n+1)/2}$ for $n>1$. For each integer $k\geq 0$ determine the number of subscripts n satisfying the conditions $2^{k+1}>n\geq 2^k$, $a_n=0$. Note: $\lfloor n/2\rfloor$ denotes the biggest integer not bigger than n/2.(Poland 1997-3rd)

160. The sequences $(a_n),(b_n),(c_n)$ are given by the conditions: $a_1=4$, $a_{n+1}=a_n(a_n-1)$, $2^{b_n}=a_n$, $e^{n-c_n}=b_n$ for n=1,2,3,.... Prove that the sequence (c_n) is bounded.(Poland 1998-1st)

161. The Fibonacci (F_n): $F_0=F_1=1$, $F_{n+2}=F_{n+1}+F_n$ for $n\geq 0$. Determine all pairs (k,m) of integer, with $m>k\geq 0$, for which the sequence (x_n) defined by $x_0=\frac{F_k}{F_m}$, $x_{n+1}=1$ for $x_n=1$, $x_{n+1}=\frac{2x_n-1}{1-x_n}$ for $x_n\neq 1$contains the number 1.(Poland 1998- 3rd)

162. Prove that the sequence (a_n) defined by; $a_1=1$; $a_n=a_{n-1}+a_{\lfloor n/2\rfloor}$ for n=2,3,4,.... contains infinitely many integers divisible by 7. Note: $\lfloor n/2\rfloor$denotes the biggest integer not bigger than n/2.(Poland 1998-3rd)

163. Let $x_1>0$ be a given real number. The sequence (x_n) defined by the formula: $x_{n+1}=x_n+\frac{1}{x_n^2}$ for n=1, 2, 3,........Prove that the limit $\lim_{n\to\infty}\frac{x_n}{\sqrt[3]{n}}$ exists and find it. (Poland 1999-1st)

164. Let S be a sequence $n_1,n_2,...,n_{1995}$ of positive integers such that $n_1+...+n_{1995}=m<3990$. Prove that for each integer q with $m\geq q\geq m$, there is a sequence $n_{i_1},n_{i_2},....,n_{i_k}$, where $1995\geq i_k>....>i_2>i_1\geq 1$, $n_{i_1}+n_{i_2}+....+n_{i_k}=q$ and k depends on q.(Singapore 95/96)

165. Suppose the number $a_0, a_1,...,a_n$ satisfy the following conditions: $a_0=\frac{1}{2}$, $a_{k+1}=a_k+\frac{1}{n}a_k^2$ for k=0,1,....,n-1. Prove that $1-\frac{1}{n}<a_n<1$.(Singapore 96/97)

166. Let $a_1 \geq .... \geq a_n \geq a_{n+1} = 0$ be a sequence of real number. Prove that $\sqrt{\sum_{k=1}^{n} a_k} \leq \sum_{k=1}^{n} \sqrt{k}(\sqrt{a_k} - \sqrt{a_{k+1}})$. (Singapore 97/98)

167. What is the smallest tower of 100s that exceeds a tower of 100 threes? In other words, let $a_1 = 3$, $a_2 = 3^3$, and a^{n+1} is 3 to the power of a_n. Similarly, $b_1 = 100$, $b_2 = 100^{100}$ etc. What is the smallest n for which $b_n > a_{100}$? (Australian 1986)

168. Define the sequence $a_1, a_2, a_3, \ldots$ by $a_1 = 1$, $a_2 = b$, $a_{n+2} = 2a_{n+1} - a_n + 2$, where b is a positive integer. Show that $a_n a_{n+1} = a_m$ for some m. (Australian 1986)

169. The real sequence $x_1, x_2, x_3, \ldots$ is defined by $x_1 = 1$, $x_{n+1} = 1/s_n$, where $s_n = x_1 + x_2 + \ldots + x_n$. Show that $s_n > 1989$ for sufficiently large n. (Australian 1989)

170. The real sequence $x_0, x_1, x_2, \ldots$ is defined by $x_0 = 1$, $x_1 = k$, $x_{n+2} = x_n - x_{n+1}$. Show that there is only one value of k for which all the terms are positive. (Australian 1991)

171. The real sequence $x_0, x_1, x_2, \ldots$ is defined as follows. $x_0 = 1$, $x_1 = 1 + k$, where k is a positive real, $x_{2n+1} - x_{2n} = x_{2n} - x_{2n-1}$, and $x_{2n}/x_{2n-1} = x_{2n-1}/x_{2n-2}$. Show that $x_n > 1994$ for all sufficiently large n. (Australian 1994)

172. Find all infinite sequences $a_1, a_2, a_3, \ldots$, each term 1 or -1, such that no three consecutive terms are the same and $a_{mn} = a_m a_n$ for all m, n. (Australian 1999)

173. The sequence $a_1, a_2, a_3, \ldots$ has $a_1 = 0$ and $a_{n+1} = \pm(a_n + 1)$ for all n. Show that the arithmetic mean of the first n terms is always at least $-\frac{1}{2}$.(Australian 2003)

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