1st ASIAN PHYSICS OLYMPIAD
KARAWACI, INDONESIA

SOLUTIONS TO EXPERIMENTAL COMPETITION

APRIL 25, 2000

Time available : 5 hours

This set of problems consists of .... pages

Solution to Problem 1

Determination of the density of oil

Experimental Configuration

This experiment is an application of Archimedes' law. The basic experimental configuration is accordingly described by Figure 1. It is assumed in this figure that the tube is in an up-right position (perpendicular to water surface). It is also clear that the same reference point must be used for measuring the positions of water or oil surfaces.

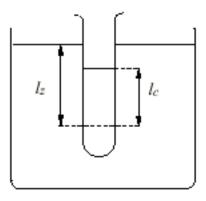Figure 1: Experimental configuration and experimental quantities to be measured.

In order to apply the law accurately to the experiment, one needs to express the precise volume occupied by the liquid inside the tube, and the volume of water displaced outside the tube. For that purpose, more detailed annotation must be introduced on the dimensional features of the test tube as shown in Figure 2.
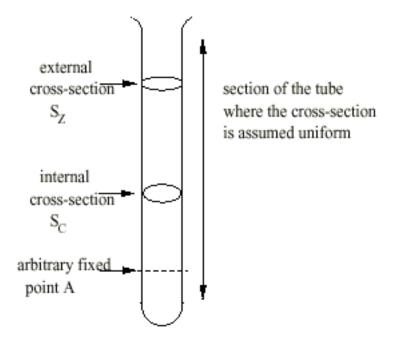
Figure 2: Specification of test tube.

Theoretical formulation

The complete listing of notations to be used in the theoretical formulation along with their corresponding definitions are given below

Sc = internal cross-section of the tube above point A
Sz = external cross-section of the tube above point A
Vo = internal volume of the test tube below point A
Ve = external volume of the test tube below point A
   = Vo volume of the glass below point A

lz = distance between point A and the water surface outside the tube
lc = distance between point A and the liquid surface inside the tube
rc = density of the liquid inside the tube
   = ρw for water
   = ρo for oil
M = mass of the empty test tube

At equilibrium, the buoyancy or the Archimedes force F_A is equal to the total weight W of the test tube including the liquid inside it. Referring to Figure 1 and 2 as well as the Notations listed above, we are led to the following expressions:

$$F_A = \left(V_e + S_z l_z\right)\rho_w g$$

$$W = \left(M + V_0 \rho_c + S_c l_c \rho_c\right)g$$

The equilibrium condition specified by $F_A = W$ implies

$$\left(V_e + S_z l_z\right)\rho_w = M + \left(V_0 + S_c l_c\right)\rho_c$$

This equation can be put into the form:

$$l_z = C + D l_c$$

where

$$D = \frac{\rho_c S_c}{\rho_w S_z}$$

$$C = \frac{M + V_0 \rho_c - V_e \rho_w}{\rho_w S_z}$$

Since the coefficient D does not depend on the zero point of l_z and l_c, the reference point A in this experiment can be chosen at some convenient point on the tube within its length of uniform cross-section as implied by the above formulation.

Measurements

In the first part of the experiment, water is used as the liquid filling the test tube to various levels corresponding to different sets of values for the pair lz and lc. Plotting lz as a function of lc on the graph paper leads to the determination of D_1,

$$D_1 = \frac{S_c}{S_z}$$

since $\rho_c = \rho_\omega$ in this case.

The same measurements are repeated in the second part, replacing water with oil for the liquid inside the tube. The result is the then given by

$$D_2 = \frac{\rho_0 S_c}{\rho_w S_z}$$

Equation Sc Sz from the two equations results in the relation

$$\rho_0 = \left(\frac{D_2}{D_1} \right) \rho_w$$

Experimental result

The experimental results consist of two parts. The results shown in Table 1 and 2 were obtained by using water and the oil as the filling liquids respectively.

Table 1: Data from experiment 1 (water).

distance from the bottom	lc (cm)	lz (cm)
2.5	3.7	11.7
2.0	4.5	12.3
1.7	4.9	12.6
1.4	5.2	12.9
1.3	5.3	13.0
1.0	5.7	13.3

The value of D_1 determined from the slope of the plot in Figure 3 is
$$D_1 = 0.8091$$

Table 2: Data from experiment 2 (parafin).

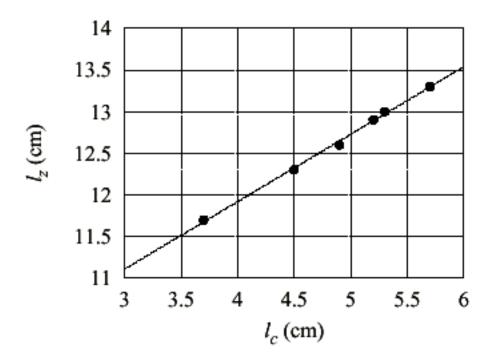| distance from the bottom	lc (cm)	lz (cm)
1.8	5.7	12.5
1.7	6.0	12.6
1.5	6.0	12.8
1.3	6.4	13.0
1.0	6.8	13.3
0.8	7.2	13.5

Figure 3: A plot of l_z vs l_c from data in Table 1.

The value of D_2 determined from the slope of the plot in Figure 4 is

$$D_2 = 0.6865$$

The final result for ρ_0 & $\Delta\rho_0$ are

$$\rho_0 = 0.8484$$
$$\Delta\rho_0 = 0.04\%$$

Remarks

1. For typical test tube, the ratio *Sc/Sz* is about 0.8 instead of 1.

2. All water and liquid surface positions to be measured must lie within the length of the tube with uniform cross-section.

3. For the determination of D_1 and D_2, one should try to get more than 2 data points and draw the line with the best fit.

4. The test tube should be dried before measurement with the oil.

5. The most crucial problem in this experiment is how to get enough data (more than two data points) with the narrowly limited range of 2 - 2.5 cm available for *lz* variation.

Suggested Grading Scheme

Theoretical Part

1. Statement of the equilibrium condition *FA* *W*   (1.0 p)
2. Expression for *FA*   (1.0 p)
3. Expression for *W*   (1.0 p)
4. Expression for *lz* as a function of *lc* including expression for *C & D*   (1.5 p)
5. Expression of the formula $\rho_0 = \rho_w \left(D_2 / D_1 \right)$ (1.5 p)

Experimental Part

1. Description of experiment detailing   (3.0 p)
    the experimental configuration   (1.0 p)
    quantities to be measured   (1.0 p)
    procedure of measurement   (1.0 p)

2. Measurement result of *lz- lc* for water (more than 2 data points)   (2.0 p)

3. Measurement result of *lz- lc* for oil (more than 2 data points)   (2.0 p)
4. Determination of D_1 and D_2 (1.0 p)
5. Determination of ρ_0 (1.0 p)
6. Accuracy of the value of ρo   (max 3.0 p)
    within 5% of the real value ρ_0 0.84 g/cm^3 (3.0 p)
    between 5% and 10% of the real value   (2.0 p)
7. Estimation of uncertainties or errors   (2.0 p)

Solution to Problem 2

Determination of Stefan-Boltzmann constant

Theoretical Consideration

According to the theory of electromagnetic radiation of solids, the polished aluminum cylinder, which can be regarded as an ideal reflector, does not absorb nor emit any radiation. On the other hand, the same cylinder covered by a thin layer of candle's soot is assumed to behave as an ideal black body, which is a perfect absorber and emitter of thermal radiation.

Therefore, the hot polished cylinder is expected to lose its thermal energy by means of non-radiative mechanism, such as thermal conductivity and convection of surrounding air. In contrast, the hot blackened cylinder will lose its thermal energy by an additional process of thermal radiation according to Stefan-Boltzmann law.

Based on the different physical processes described above, 3 different methods of experiment can be formulated as follows:

1. Method of constant temperature

   Assume that the cylinder is heated to the same temperature T when it is unblackened (polished) and when it is blackened by the soot. The difference in the measured electric power needed to reach that same equilibrium temperature must be equal to power loss due to radiative process. In other words,

   $$P_r(T) = P_t(T) - P_n(T)$$

   where:

   P_r (T)= power loss of the blackened cylinder due to thermal radiation
   P_t (T)= total power loss of the blackened cylinder at T

   P_n (T)=power loss of the polished cylinder at T due to nonradiative processes

   Assuming the same P_n T = in both cases (polished and blackened), one obtains
   $$\sigma = \frac{P_t(T) - P_n(T)}{S(T^4 - T_0^4)}$$

where *To* is the surrounding (or room) temperature

Alternatively, although less accurately, other methods may also be formulated by explicitly assuming that *Pn* is proportional to $(T - T_0)$, namely

$$P_n(T) = k(T - T_0)$$

where *k* is a constant independent of T. On the basis of this relation, one can formulate the following two methods for the determination of σ.

2. Method of constant power

In this method, the power of heating *P* is kept the same in both cases. Let the temperatures reached in equilibrium for the polished and blackened cylinder be denoted by *Tp* and *Tb* respectively. Then,

$$P = k\left(T_p - T_0\right)$$
$$P = k\left(T_b - T_0\right) + P_r\left(T_b\right)$$

Eliminating *k* yields

$$P_r(T_b) = \frac{\left(T_p - T_b\right)P}{\left(T_p - T_0\right)}$$

Equating this to the radiative power expression of the Stefan-Boltzmann law, we Obtain

$$\sigma = \frac{\left(T_p - T_b\right)P}{S(T_b^4 - T_0^4)\left(T_p - T_0\right)}$$

3. Method of two temperatures
In this case, the measurements are performed for the blackened cylinder only, but at two equlibrium temperatures T_1 and T_2. Let the heating powers required to reach
T_1 and T_2 be P_1 and P_2 respectively. Then we have

$$P_1 = k\left(T_1 - T_0\right) + \sigma S\left(T_1^4 - T_0^4\right)$$
$$P_2 = k\left(T_2 - T_0\right) + \sigma S\left(T_2^4 - T_0^4\right)$$

Again, eliminating *k* from the two equations above leads directly to the following

Expression

$$\sigma = \frac{(T_2 - T_0) - (T_1 - T_0)P_2}{S\left[\left(T_1^4 - T_0^4\right)(T_2 - T_0) - \left(T_2^4 - T_0^4\right)(T_1 - T_0)\right]}$$

Remarks

The formulation of the first experimental method requires the insurance of the same T in both cases. Since P is proportional to T^4, a small difference in T determined in two cases will result in great error. It is, however, not easy to satisfy the requirement mentioned above. One way of overcoming this difficulty is to measure the power Pt for heating up the blackened cylinder at two temperatures in the vicinity of the temperature reached by the unblackened cylinder, and interpolate the value of Pt at the right T.

It is also worth noting that due to the sensitivity of the measurement, a slight change in the surrounding of the cylinder is likely to affect the result significantly. The environment must therefore be kept constant during the experiment.

Experimental Configuration

The experimental set-up is described in Figure 1. The heater is mounted on a porcelain base, and it is connected with a power supply and the measuring meters. The heater is entirely enclosed by the hollow cylinder which sits also on the same porcelain plate during the measurement. The thermocouple is permanently attached to the cylinder and connected to an mV-meter for the determination of the temperature by using a table listing the characteristics of the thermocouple. The size of the cylinder is 60 mm by length and 12.5 mm by its external diameter, leading to a surface area of S=24.8 cm^2. The wall of the cylinder is about 1 mm thick and the thickness of its base is about 3 mm. All electrical measuring meters are digital instruments.

The power supplied to the heater must be measured separately instead of being read off the power supply display panel, because the resistance of the heater varies somewhat with temperature. The reading of V and I should be done at thermal equilibrium between the cylinder and its surrounding, which will be reached in about 25-30 minutes. In order to avoid undesirable effects from the surrounding, the whole system should be kept at a distance from other objects in the laboratory.

Results of measurement
In a set of experiments performed at room temperature of 298.8 K, the results obtained
are represented by the sample data given in Table 1.
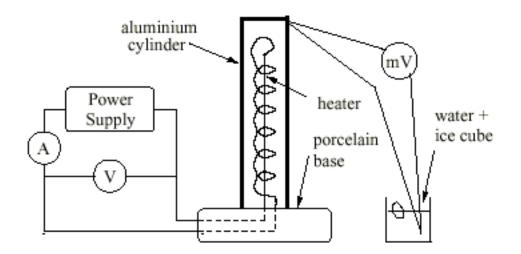

Figure 1: Experimental set-up.

Table 1: The values of σ found in a set of three measurements

Code name for the data	Surface condition during measurement	Data		
		V	A	$T\,K$
a	polished	9.8	1.50	485.5
b	blackened	9.8	1.50	433.5
c	blackened	11.9	1.82	485.5

Discussion

While the last two methods are supposed to be less accurate than the first one, this is not
always confirmed by the experimental results, as the control of experimental condition is
not perfect. The major factors affecting the accuracies of the experimental results are
enumerated and discussed as follows:

1. The cylinder is not necessarily an ideal reflector when it surface is polished, nor is it an ideal black body when its surface is blackened by the candle's soot. In other words, the absorption coefficient is likely to be larger than 0 in the first case, and less than 1 for the second case. Both of these effects leads to lower value of s.

2. The heat losses via the porcelain base are out of control. Neglecting these losses will lead to deviation of σ from its real value.

3. The resistivities of the connecting cables have been neglected also, leading to larger value of s.

Table 2: Results of σ obtained by three different methods
.

Method used	Data used	Experimental result $\sigma ex \ W\,m^{2}\,K^4$	$\sigma_{ex}\sigma$
constant T	$a+c$	$5.945 \ 10^{-8}$	1.05
constant P	$a+b$	$6.087 \ 10^{-8}$	1.07
two T ;s	$b+c$	$5.386 \ 10^{-8}$	0.95

4. The assumption of equal non-radiative loss for the case with polished and blackened surfaces is at best an approximation. For instance, the difference between thermal conductivity of the soot and that of aluminium is neglected in this experiment, leading to lower value of s. The equality will also be violated due to uncontrollable heat losses via the porcelain base.

5. The influences of air convection in the surrounding of the cylinder due to motions of the experimentator and other objects are also possible sources of errors.

Suggested Grading Scheme

Theoretical part

1. Statement of non-radiative nature of the thermal energy loss in the case of cylinder with polished surface (1.0 p)
2. Recognition of non-radiative as well as radiative contributions to energy loss in the case of cylinder with blackened surface (1.5 p)
3. Assumption of equal non-radiative losses in both cases for the same final equilibrium temperature T (2.0 p)
4. Derivation of formula for σ (2.5 p)

Remarks

In case the participants employ the second or the third method, the first three item in the grading scheme for the theoretical part should be accordingly adjusted and combined as folllows.

1. Statements on radiative and non-radiative processes of heat trans-fer (2.5 p)
2. Assumption of linear dependence of non radiative loss on tem-perature difference (2.0 p)

Experimental part

1. Description of the experimental set-up (4.0 p)
   The wiring of measuring instruments (1.5 p)
   Method and procedure of measurement (1.5 p)
   The quantities to be measured (1.0 p)

2. Results of measurement (data of V A and T) (3.0 p)
3. Value of σ (1.0 p)
4. Accuracy of value of σ (max 3.0 p)
   within 10% of the real value
   $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} (3.0 p)
   between 10% and 20% of the real value (2.0 p)
5. Estimation of uncertainties or errors (2.0 p)